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Exact travelling wave solutions for the local fractional two-dimensional Burgers-type equations

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ABSTRACT

In this paper, a family of local fractional two-dimensional Burgers-type equations (2DBEs) is investigated. The local fractional Riccati differential equation method is proposed here for the first time. The travelling wave transformation of the non-differentiable type is presented. The non-differentiable exact travelling wave solutions for the problems are obtained. The present methodology is shown to provide a useful approach to solve the local fractional nonlinear partial differential equations (LFNPDEs) in mathematical physics.

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1. Introduction

In recent years, a large number of methods were developed to find the exact solutions for the linear and nonlinear partial differential equations. Among them, there are Adomian decomposition method (ADM) [1,2], variational iteration method (VIM) [3,4], homotopy analysis method (HAM) [5,6], Riccati expansion method (RExM) [7] and Riccati equation method (REqM) [8] etc.

The two-dimensional (2D) Burgers-type equations (BEs) were used to describe the nonlinear phenomena in flow. These developments attracted interest by researchers in many different fields to find the methodologies to obtain the solutions for the BEs. For example, the lattice Boltzmann method (LBM) was proposed to find the numerical solution for the BE [9]. The Chebyshev spectral collocation method was reported for solving the BEs [10].

Recently, the fractional-calculus analogue of the BEs was presented in [11] and some methods for finding its solution were developed in [12–14]. For example, the parametric spline functions technology (PSFT) was used to solve the time-fractional BEs [12]. The solution for the space- and time-fractional BEs was discussed by using the ADM [13] and VIM [14], respectively.

More recently, the local fractional one-dimensional Burgers-type equation was proposed in [15,16]. The local fractional calculus (LFC) was used to model a large number of the complex problems involving fractal engineering (see [16,17]). The many technologies, e.g., Two-dimensional extended differential transform (TEDT) [18], VIM [19] and Exp-function

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Table 1
The basic operations of the LFDs of the special functions.

Special functions	LFDs
$E_\varepsilon\,(\zeta^\varepsilon)$	$E_\varepsilon\,(\zeta^\varepsilon)$
$\sin_\varepsilon\,(\zeta^\varepsilon)$	$\cos_\varepsilon\,(\zeta^\varepsilon)$
$\cos_\varepsilon\,(\zeta^\varepsilon)$	$-\sin_\varepsilon\,(\zeta^\varepsilon)$

Table 2
The special functions defined on fractal sets.

Special functions	Formulas
$\tan_\varepsilon\,(\zeta^\varepsilon)$	$\dfrac{E_\varepsilon\left(i^\varepsilon \zeta^\varepsilon\right)-E_\varepsilon\left(-i^\varepsilon \zeta^\varepsilon\right)}{i^\varepsilon\left(E_\varepsilon\left(i^\varepsilon \zeta^\varepsilon\right)+E_\varepsilon\left(-i^\varepsilon \zeta^\varepsilon\right)\right)}$
$\cot_\varepsilon\,(\zeta^\varepsilon)$	$\dfrac{i^\varepsilon\left(E_\varepsilon\left(i^\varepsilon \zeta^\varepsilon\right)+E_\varepsilon\left(-i^\varepsilon \zeta^\varepsilon\right)\right)}{E_\varepsilon\left(i^\varepsilon \zeta^\varepsilon\right)-E_\varepsilon\left(-i^\varepsilon \zeta^\varepsilon\right)}$
$\tanh_\varepsilon\,(\zeta^\varepsilon)$	$\dfrac{E_\varepsilon\left(\zeta^\varepsilon\right)-E_\varepsilon\left(-\zeta^\varepsilon\right)}{E_\varepsilon\left(\zeta^\varepsilon\right)+E_\varepsilon\left(-\zeta^\varepsilon\right)}$
$\coth_\varepsilon\,(\zeta^\varepsilon)$	$\dfrac{E_\varepsilon\left(\zeta^\varepsilon\right)+E_\varepsilon\left(-\zeta^\varepsilon\right)}{E_\varepsilon\left(\zeta^\varepsilon\right)-E_\varepsilon\left(-\zeta^\varepsilon\right)}$

method [20], were developed to find the solutions for local fractional partial differential equations (LFNPDEs). By using the result in [21], the main aim of this paper is to propose the local fractional Riccati differential equation method to solve the local fractional 2DBEs (see also [22–30] for various other related developments involving fractional differential equations). The structure of the paper is given as follows. In Section 2, the basic definition of the local fractional calculus and the solutions of the local fractional Riccati differential equations are presented. In Section 3, the local fractional Riccati differential equation method is proposed. The travelling wave solutions for the local fractional 2DBEs are discussed in Section 4. Finally, the conclusion is outlined in Section 5.

2. Introduction

In this section, the concepts of local fractional derivatives (LFD) and local fractional partial derivatives (LFPDs) of the non-differentiable functions and the solutions for the local fractional Riccati differential equations (LFRDE) are given.

2.1. Theory of LFC

Suppose that $\mathbb{C}_\varepsilon(x, y)$ denotes a set of the non-differentiable functions (NFs) with the fractal dimension ε $(0 < \varepsilon < 1)$, see [16].

Definition 1. Suppose that $\varXi\,(\zeta) \in \mathbb{C}_\varepsilon(x, y)$. Then LFD of $\varXi\,(\zeta)$ of order ε $(0 < \varepsilon < 1)$ at the point $\zeta = \zeta_0$ is given by (see [16]):

$$D^{(\varepsilon)}\varXi\,(\zeta_0) = \frac{d^\varepsilon \varXi\,(\zeta_0)}{d\zeta^\varepsilon} = \frac{\Delta^\varepsilon\,(\varXi\,(\zeta) - \varXi\,(\zeta_0))}{(\zeta - \zeta_0)^\varepsilon}, \tag{1}$$

where

$$\Delta^\varepsilon\,(\varXi\,(\zeta) - \varXi\,(\zeta_0)) \cong \Gamma\,(1 + \varepsilon)\,\Delta\,[\varXi\,(l) - \varXi\,(l_0)]. \tag{2}$$

The basic operations of LFD are given as follows (see [16,18]):

(T1) $D^{(\varepsilon)}\,[\varXi_1\,(\zeta) \pm \varXi_2\,(\zeta)] = D^{(\varepsilon)}\varXi_1\,(\zeta) \pm D^{(\varepsilon)}\varXi_2\,(\zeta)$,
(T2) $D^{(\varepsilon)}\,[\varXi_1\,(\zeta)\,\varXi_2\,(\zeta)] = \left[D^{(\varepsilon)}\varXi_1\,(\zeta)\right]\varXi_2\,(\zeta) + \varXi_1\,(\zeta)\left[D^{(\varepsilon)}\varXi_2\,(\zeta)\right]$,
(T3) $D^{(\varepsilon)}\,[\varXi_1\,(\zeta)\,/\varXi_2\,(\zeta)] = \left\{\left[D^{(\varepsilon)}\varXi_1\,(\zeta)\right]\varXi_2\,(\zeta) - \varXi_1\,(\zeta)\left[D^{(\varepsilon)}\varXi_2\,(\zeta)\right]\right\}/\varXi_2^2\,(\zeta)$, provided $\varXi_2\,(\zeta) \neq 0$.

The LFDs of the special functions defined on fractal sets (see [16]) are illustrated in Table 1.
The special functions defined on fractal sets (see [16]) are structured in Table 2.
It is necessary that the above special function defined on fractal sets used in this paper were addressed in [16].

Definition 2. The local fractional partial derivative (LFPD) of $\varXi_\varepsilon\,(\theta, \vartheta)$ of order ε with respect to ϑ is defined by [16,18–21]:

$$\varXi_{\vartheta,\varepsilon}^{(\varepsilon)}\,(\theta, \vartheta_0) = \frac{\partial^\varepsilon \varXi_\varepsilon\,(\theta, \vartheta)}{\partial \vartheta^\varepsilon}\bigg|_{\vartheta=\vartheta_0} = \lim_{\vartheta \to \vartheta_0} \frac{\Delta^\varepsilon\,(\varXi_\varepsilon\,(\theta, \vartheta) - \varXi_\varepsilon\,(\theta, \vartheta_0))}{(\vartheta - \vartheta_0)^\varepsilon}, \tag{3}$$

provided that the lime exists, where

$$\Delta^\varepsilon\,(\varXi_\varepsilon\,(\theta, \vartheta) - \varXi_\varepsilon\,(\theta, \vartheta_0)) \cong \Gamma\,(1 + \varepsilon)\,\Delta\,(\varXi_\varepsilon\,(\theta, \vartheta) - \varXi_\varepsilon\,(\theta, \vartheta_0)). \tag{4}$$