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Abundant lump-type solutions of the Jimbo–Miwa equation in $(3+1)$-dimensions

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ABSTRACT

Based on the Hirota bilinear form of the (3+1)-dimensional Jimbo–Miwa equation, ten classes of its lump-type solutions are generated via Maple symbolic computations, whose analyticity can be easily achieved by taking special choices of the involved parameters. Those solutions supplement the existing lump-type solutions presented previously in the literature.

1. Introduction

Integrable equations possess Hirota bilinear forms, and among important examples of such equations are the Korteweg–de Vries (KdV) equation, the Boussinesq equation, the Kadomtsev–Petviashvili (KP) equation, the BKP equation and the Toda lattice equation [1]. All those integrable equations have exponentially localized solutions—soliton solutions [2]. It is the Hirota bilinear formulation that plays a key role in generating soliton solutions, but some intelligent guesswork is often required [3].

Besides soliton solutions, there exist rational solutions to nonlinear partial differential equations, certainly to integrable equations (see, e.g., [4,5]). Particularly important are rationally localized solutions in all directions of space, called lump solutions, and examples of lump solutions are found for many interesting nonlinear equations arising from physically relevant situations, which contain the KP equation I [6,7], the three-dimensional three-wave resonant interaction [8], the BKP equation [9,10], the Davey–Stewartson equation II [11] and the Ishimori-I equation [12]. In particular, the KP equation I of the following form:

$$(u_t + 6uu_x + u_{xxx})_x - 3u_{yy} = 0, \tag{1.1}$$

has the lump solution [6]:

$$u = 4\frac{-[x+ay+3(a^2-b^2)t]^2 + b^2(y+6at)^2 + 1/b^2}{\{[x+ay+3(a^2-b^2)t]^2 + b^2(y+6at)^2 + 1/b^2\}^2}, \tag{1.2}$$

where a and b are real free parameters. Rogue wave solutions, which draw a big attention of mathematicians and physicists in the international research community, are a particularly important kind of lump or lump-type solutions, and such

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solutions, usually with rational function amplitudes, could be used to describe interesting nonlinear wave phenomena in both oceanography [13] and nonlinear optics [14]. Lump or lump-type solutions to nonlinear partial differential equations present an interesting research question for us, and Hirota bilinear forms will be a good basis for carrying out research on such solutions.

General rational solutions to integrable equations have been considered within the Wronskian formulation, the Casoratian formulation and the Grammian or Pfaffian formulation (see [1,2]). Typical examples include the KdV equation and the Boussinesq equation in (1+1)-dimensions, the KP equation in (2+1)-dimensions, and the Toda lattice equation in (0+1)-dimensions (see, e.g., [15–18]). A few new attempts have also been made to look for rational solutions to the non-integrable (3+1)-dimensional KP I [19,20] and KP II [21] by direct analytical approaches, for example, the tanh-function method, the $\frac{G'}{G}$-expansion method and symbolic computations (see, e.g., [22–26]). There is a link of rational solutions between the (3+1)-dimensional KP II and the good Boussinesq equation [21], bilinear Bäcklund transformations are applied to rational solutions to (3+1)-dimensional generalized KP equations (see, e.g., [27]), and there exist some direct searches for rational solutions to generalized bilinear equations (see, e.g., [24,16,26]). Moreover, there are studies on rational solutions to nonlinear equations by the Exp-function method without using Hirota bilinear forms [28,29].

In this paper, we would like to consider the Jimbo–Miwa equation in (3+1)-dimensions [30] and generate ten classes of its lump-type solutions by Maple symbolic computations, based on the studies on lumps to (2+1)-dimensional equations (see, e.g., [7]). The resulting lump-type solutions supplement the existing lump-type solutions in the literature (see, e.g., [31]). The (3+1)-dimensional Jimbo–Miwa equation possesses a Hirota bilinear form, and thus, we will search for positive quadratic function solutions of the corresponding (3+1)-dimensional bilinear Jimbo–Miwa equation. A few concluding remarks will be given finally in Section 3.

2. Abundant lump-type solutions

The (3+1)-dimensional Jimbo–Miwa equation reads [30]

$$P_{JM}(u) := u_{xxxy} + 3u_yu_{xx} + 3u_xu_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{2.1}$$

called the Jimbo–Miwa equation in [32]. The equation is among the entire KP hierarchy [30] and completely defined by a Hirota bilinear equation

$$\begin{aligned} B_{JM}(f) &:= (D_x^3D_y + 2D_tD_y - 3D_xD_z)f \cdot f \\ &= 2(f_{xxxy}f - f_yf_{xxx} - 3f_xf_{xxy} + 3f_{xx}f_{xy} + 2f_{yt}f - 2f_yf_t - 3f_{xz}f + 3f_xf_z) = 0, \end{aligned} \tag{2.2}$$

under the transformation between f and u:

$$u = 2(\ln f)_x. \tag{2.3}$$

This is also a characteristic transformation adopted in Bell polynomial theories of soliton equations (see, e.g., [33,34]) and actually we have

$$P_{JM}(u) = \frac{f(B_{JM}(f))_x - 2f_xB_{JM}(f)}{f^3}. \tag{2.4}$$

Therefore, when f solves the bilinear Jimbo–Miwa equation (2.2), $u = 2(\ln f)_x$ will present a solution to the (3+1)-dimensional Jimbo–Miwa equation (2.1).

The Hirota perturbation technique allows us to present one- and two-soliton solutions [32] and dromion-type solutions [35], and the Exp-function method and the transformed rational function method generates many traveling wave solutions [36,37]. A direct computation also shows that the Jimbo–Miwa equation (2.1) has the following polynomial solutions [37]:

$$u = a_0 + a_1x + a_2y + a_3z + a_4t + a_5xz + a_6xt + a_7yz + \frac{3}{2}a_5yt + a_8zt, \tag{2.5}$$

where $a_i,\ 0 \le i \le 8$, are arbitrary parameters.

In what follows, we focus on computing lump-type solutions to the (3+1)-dimensional Jimbo–Miwa equation (2.1) through carefully searching for positive quadratic function solutions to the bilinear Jimbo–Miwa equation (2.2) with symbolic computations.

We apply the computer algebra system Maple to look for quadratic function solutions to the (3+1)-dimensional bilinear Jimbo–Miwa equation (2.2). A direct Maple symbolic computation starting with

$$\begin{cases} f = g^2 + h^2 + a_{11}, \\ g = a_1x + a_2y + a_3z + a_4t + a_5, \\ h = a_6x + a_7y + a_8z + a_9t + a_{10}, \end{cases} \tag{2.6}$$

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