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Workload smoothing in simple assembly line balancing

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ABSTRACT

This paper considers a simple assembly line balancing problem with fixed number of workstations and prespecified cycle time. Our objective is to minimize the sum of the squared deviations of the workstation loads around the cycle time, hence maintain workload smoothing. We develop several optimality properties and bounding mechanisms, and use them in our branch and bound algorithm. The results of our computational study reveal that our branch and bound algorithm is capable of solving medium sized problem instances in reasonable times.

1. Introduction

The assembly lines are used extensively in mass production systems to produce high quantity standardized products. They consist of a number of serially connected workstations and a material handling system connecting the workstations. The repetitive assembly operations (tasks) are performed on the workstations as the product flows along the line. Becker and Scholl (2006) classify the assembly lines into three types: single model, multi-model and mixed-model. We consider a single model line where the assembly line is dedicated to the production of one single product.

Assembly line balancing (ALB) is the act of assigning the tasks to the workstations by optimizing the pre-specified objective function without violating the precedence constraints. ALB that produces a single model is referred to as a simple assembly line balancing problem (SALBP).

Based on the objective functions the SALBP can be categorized as Type I, Type II and Type III (Baybars, 1986; Scholl, 1999; Scholl and Becker, 2006). Type I problems minimize the number of workstations given a prespecified cycle time. The cycle time is defined as the time between the completion times of two successive product units. Type II problems minimize the cycle time, hence maximize the production rate, given a prespecified number of workstations. Type III problems maximize the workload smoothing (balancing), hence try to attain similar workstation loads. Boysen et al. (2007) and Battaia and Dolgui (2013) give the comprehensive reviews on the assembly line balancing area. We hereafter refer to Type III problems as workload smoothing problems.

The importance of the workload smoothing problems has been mentioned in many studies. As stated in Rachamadugu and Talbot (1991) workload smoothing brings the notion of equity in manual assembly lines. Uneven distribution of the workloads is viewed unfair which might trigger different pays of the workers. The empirical study of Smunt and Perkins (1985) shows that in particular, the long assembly lines should balance the workload among the workstations en route to maximizing the production rate. Groover (2007) mentions that the unequal distribution of tasks does not only create workload imbalance between workstations, but also increase the ergonomic risks. Otto and Scholl (2011) develop the ergonomic risk estimation methods and evaluate the trade-off between the number of workstations and decrease in ergonomic risks. They mention that the unbalanced workload decreases the remaining lifetime of the machines thereby increasing their breakdown probabilities and chance of reaching the target production rate.

Despite the importance of the workload smoothing in assembly lines, the related research is quite scarce. All studies propose heuristics procedures, some noteworthy of which are Rachamadugu and Talbot (1991), Ponnambalam et al. (2000), Mozdgir et al. (2013), Rachamadugu and Talbot (1991) study the mean absolute deviation of the workstation loads from a target load and propose an iterative heuristic procedure. Ponnambalam et al. (2000) propose a multi-objective genetic algorithm for different performance measures such as the number of workstations and the difference between the maximum workstation time and workloads of the each workstation. Mozdgir et al. (2013) propose evolutionary computation based

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method on the differential evolution algorithm to minimize the sum of the squared differences between the cycle time and each workstation load, so called workstation smoothness index.

Eswaramoorthi et al. (2012) and Wenping et al. (2013) consider workload smoothing index. Eswaramoorthi et al. (2012) present a two stage heuristic procedure based on the concepts of the COMSOAL (computer method of sequencing operations for assembly lines) algorithm. Wenping et al. (2013) propose a memetic algorithm.

In this study, we consider workload smoothing problem in single model assembly lines. Our main motivation is the practical importance of the problem and scarcity of the related research.

We assume the line is already configured with a fixed number of workstations. Moreover we assume that the cycle time is pre-specified, i.e., the maximum workload is below a predefined amount, hence the predefined target production rate is reached. We assign the tasks to the workstations so as to minimize the total squared deviation around the defined cycle time. We show that our objective is equivalent to minimizing the sum of the squared workstation loads. We give the linear equivalence of our objective function and then define the associated mixed integer linear programming model. We propose a branch and bound algorithm that uses powerful bounding and reduction mechanisms.

To the best of our knowledge, our study is the first optimization attempt for the workload smoothing problem, in assembly lines.

The rest of the paper is organized as follows. Section 2 defines the problem and gives its mathematical model. In Section 3, we settle the complexity of the problem and present the optimality properties. The branch and bound algorithm is discussed in Section 4. Section 5 reports the results of our computational study. Section 6 concludes the study and discusses some future research directions.

2. Problem definition and the model

Consider N tasks and K workstations. All tasks can be performed in all workstations, and all workstations are equipped identically. We determine the assignment of the tasks to each workstation so as to minimize the sum of the squared deviation of the workload from the cycle time, hence to balance the workload among the workstations.

The environment is deterministic and static, i.e., the parameters (task times, precedence relations) are known with certainty and not subject to any change. All workstations are reliable, i.e., available at all times.

We use the following parameters and decision variables to present our model.

Parameters:

t_i : processing time of task i $\quad i\epsilon\{1,\ldots,N\}$

(u, v): precedence relations among tasks, task u should immediately precede task v

IP: set of all immediate predecessors

C: cycle time

Decision Variables:

$$x_{ik} = \begin{cases} 1 & \text{if task i is assigned to workstation } k \\ 0 & \text{Otherwise} \end{cases} \qquad i\epsilon\{1,\ldots,N\} \text{ and } k\epsilon\{1,\ldots,K\}$$

W_k: sum of the task time assigned to the workstation k, i.e., workload of workstation k

Our objective function is $\sum_{k=1}^{K}(C-W_k)^2$ in which we minimize the sum of the squared differences between the 'cycle time' and 'workstation load'. $\sum_{k=1}^{K}(C-W_k)^2$ is also referred to as flow index and smoothness index, in the literature.

Theorem 1 below shows that minimizing $\sum_{k=1}^{K}(C-W_k)^2$ is equivalent to minimizing total squared load over all workstations.

Theorem 1. *Minimizing* $\sum_{k=1}^{K}(C-W_k)^2$ *is equivalent to minimizing* $\sum_{k=1}^{K} {W_k}^2$.

Proof.

$$Z = \sum_{k=1}^{K}(C-W_k)^2 = \sum_{k=1}^{K}(C^2 - 2CW_k + {W_k}^2)$$

$$= \sum_{k=1}^{K} C^2 - 2C\sum_{k=1}^{K} W_k + \sum_{k=1}^{K} {W_k}^2$$

$$\sum_{k=1}^{K} W_k = \sum_{i=1}^{N} t_i \text{ follows } C^2 - 2C\sum_{i=1}^{N} t_i + \sum_{k=1}^{K} {W_k}^2$$

Recall that the first two terms of Z are constant, hence irrelevant for optimization. This follows Min $Z \equiv$ Min $\sum_{k=1}^{K} {W_k}^2$ □

Theorem 1 reduces our objective function to:

$$\text{Min } Z = \text{ Min} \sum_{k=1}^{K} {W_k}^2$$

We let R_k be the set of tasks assigned to workstation k and rewrite the objective function as:

$$Z = \sum_{k=1}^{K} {W_k}^2 = \sum_{k=1}^{K}\left[\sum_{i\in R_k} t_i\, x_{ik}\right]^2$$

$$= \sum_{k=1}^{K}\sum_{i\in R_k}(t_i\, x_{ik})^2 + \sum_{k=1}^{K}\left(\sum_{i\in R_k}\ \sum_{j\in R_k,\, j\neq i} t_i\, t_j\, x_{ik}\, x_{jk}\right)$$

We define y_{ijk} variables as:

$$y_{ijk} = \begin{cases} 1 & x_{ik} = x_{jk} = 1 \\ 0 & \text{Otherwise} \end{cases} \quad i\epsilon\{1,\ldots,N\},\ j\epsilon\{1,\ldots,N\} \text{ and } k\epsilon\{1,\ldots,K\}$$

Accordingly, our objective function can be rewritten as:

$$\text{Min } Z \equiv \sum_{k=1}^{K}\left[\sum_{i} t_i^2\, y_{iik} + 2\sum_{i}\sum_{j} t_i\, t_j\, y_{ijk}\right] \tag{1}$$

We introduce the following constraint sets to support the definition of y_{ijk}.

$$y_{ijk} \geq x_{ik} + x_{jk} - 1 \qquad \forall i,\ j, k \tag{2}$$

$$y_{ijk}\epsilon\ \{0,\ 1\} \quad \forall i,\ j, k \tag{3}$$

The constraints of the problem are introduced below.

$$\sum_{i=1}^{N} t_i\, x_{ik} = W_k \quad \forall k \tag{4}$$

$$W_k \leq C \qquad \forall k \tag{5}$$

$$\sum_{k=1}^{K} k\, x_{uk} \leq \sum_{k=1}^{K} k\ x_{vk} \qquad (u,v)\epsilon IP \tag{6}$$

$$\sum_{k=1}^{K} x_{ik} = 1 \quad \forall i \tag{7}$$

$$x_{ik}\epsilon\{0, 1\} \qquad \forall i, k \tag{8}$$