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On the exact solution of the no-wait flow shop problem with due date constraints

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A B S T R A C T

This paper deals with the no-wait flow shop scheduling problem with due date constraints. In the no-wait flow shop problem, waiting time is not allowed between successive operations of jobs. Moreover, the jobs should be completed before their respective due dates; due date constraints are dealt with as hard constraints. The considered performance criterion is makespan. The problem is strongly NP-hard. This paper develops a number of distinct mathematical models for the problem based on different decision variables. Namely, a mixed integer programming model, two quadratic mixed integer programming models, and two constraint programming models are developed. Moreover, a novel graph representation is developed for the problem. This new modeling technique facilitates the investigation of some of the important characteristics of the problem; this results in a number of propositions to rule out a large number of infeasible solutions from the set of all possible permutations. Afterward, the new graph representation and the resulting propositions are incorporated into a new exact algorithm to solve the problem to optimality. To investigate the performance of the mathematical models and to compare them with the developed exact algorithm, a number of test problems are solved and the results are reported. Computational results demonstrate that the developed algorithm is significantly faster than the mathematical models.

1. Introduction

In the classical flow shop scheduling problem there is a set of n jobs that has to be processed with a predefined order of operations on m machines, and the optimal sequence of jobs on each machine with respect to some performance measure is desired. It is also common to assume that jobs have identical sequence on all machines, which is known as the permutation flow shop scheduling problem.

Considered in this paper is a flow shop scheduling problem with the makespan criterion with two additional assumptions, namely allowing no waiting time between the operations and considering due date for each job. In the no-wait flow shop scheduling problem, no waiting time is allowed between successive operations of jobs. In other words, once processing of a certain job is started, no interruption is permitted between the operations of that job. In addition to the no-wait constraint we assume that the completion of each job is associated with a due date, i.e. jobs must be completed before their due dates. Due date are among the most applicable constraints in scheduling and sequencing literature because real-world jobs are usually accompanied by a deadline for completion [27]. In this paper, it is assumed that all the jobs are ready at time zero (all release dates are zero) and no preemption or interruption in the process of operations is allowed. According to the conventional three-field notation of the scheduling problems [21], the problem can be designated as $F|nwt, d_j|C_{\max}$.

It has been shown by Wismer [62] and Bonney and Gundry [9] that the no-wait flow shop problem with makespan performance measure ($F|nwt|C_{\max}$) can be reduced to the asymmetric travelling salesperson problem (ATSP). Based on this relation between no-wait flow shop and ATSP, King and Spachis [32] developed a heuristic to solve no-wait flow shop problems. It is proved by Lenstra and Kan [33], using a reduction from the directed Hamiltonian path problem, that the problem $F|nwt|C_{\max}$ with m machines is NP-hard, when $m \geq 4$. They also showed that with $m=2$ the problem is solvable in polynomial time. The NP-hardness of the case with $m=3$ is shown by Röck [49] using a reduction from three-dimensional matching (3DM) problem. Röck [49] summarizes the complexity of a group of similar problems. Since $F|nwt|C_{\max}$ is a special case of$F|nwt, d_j|C_{\max}$ it can be con-

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cluded that $F|nwt, d_j|C_{\max}$ with at least three machines is also NP-hard in the strong sense.

Industrial applications mentioned in the literature for $F|nwt, d_j|C_{\max}$ include chemical industries [47], food industries [25], steel production [62], pharmaceutical industries [45], and production of concrete products [20]. Hall and Sriskandarajah [25] provide a comprehensive review of the applications of the problem.

The reputation of a company as a reliable firm will be tremendously damaged if it frequently delivers jobs after their due dates are passed (even if the number of late days is relatively small). Moreover, trust between companies will be damaged if late jobs are not frequent, but a few jobs are delivered considerably past their due dates. Note that on-time delivery of the jobs can be only one of the goals of a company. Companies can be interested in optimizing other criteria such as makespan, while avoiding late days or tardy jobs. Hence, $F|nwt, d_j|C_{\max}$ is not only an applicable problem with many real-world applications, but it is proved to be NP-hard and theoretically interesting.

The rest of the paper is organized as follows. Section 3 describes the notations used. Section 4 formulates the mathematical programming models. Section 5 describes the novel graph representation and the enumeration algorithm. Computational experiments are reported in Section 6. Section 6.3 gives concluding remarks and discusses future research directions.

2. Related work

The literature is rich with studies that develop heuristic or metaheuristic methods in order to deal with no-wait flow shop scheduling problems with or without due dates constraints. For the case of $F|nwt, d_j|\gamma$, due date constraints have been traditionally considered as soft constraints. In other words, violating due date constraints has been permitted with the objective function of minimizing a measure of the tardiness (e.g., number of tardy jobs or number of late days); tardiness measures have frequently been combined with other performance measures such as makespan, total flow time, etc.

Since no-wait flow shop problem with due date constraints is strongly NP-hard, several algorithms have been devised to deal with the problem. These efforts are reviewed in two categories, namely the heuristic or metaheuristic methods and exact methods, since both approaches can be useful depending on the size of the problems.

2.1. Heuristic and metaheuristic methods

Table 1 summarizes some of the early efforts to solve the scheduling problems with a form of due date constraints using a non-exact method. More recently, Tang et al. [56] developed a metaheuristic to deal with fuzzy due dates in a flow shop environment. Panwalkar and Koulamas [41] considered a two-machine flow shop problem with the objective of minimizing the total tardy jobs and finding a common due date for the jobs, and developed a heuristic algorithm with computational complexity of $O(n^2)$ for a special case of the problem. This algorithm was further improved to an improved $O(n\log n)$ algorithm by Ilić [28].

Aldowaisan and Allahverdi [1] considered the flow shop scheduling problem with the objective of minimizing the number of tardy jobs and proposed a number of metaheuristics to deal with the problem. Arabameri and Salmasi [4] considered the no-wait flow shop problem with the objective of minimizing the weighted earliness and tardiness penalties; they developed a MILP as well as a number of metaheuristics for the problem. The developed mathematical model of Arabameri and Salmasi [4] is very similar to the model of Samarghandi [50]; however, it lacks some of the constraints of the model of Samarghandi [50].

Pang [40] developed a genetic algorithm to deal with a two-machine no-wait flow shop problem with the objective of minimizing the maximum lateness of the jobs. Tasgetiren et al. [58] considered the no-idle permutation flow shop scheduling problem with the total tardiness criterion and proposed an artificial bee colony algorithm to deal with the problem. Liu et al. [34] proposed numerous heuristics for the no-wait flow shop problem with the objective of minimizing the total tardiness, and compared the results of the heuristics with each other.

Tari and Olfat [57] considered a flow shop problem with due date constraints and proposed a number of heuristics to minimize the total tardiness. Ebrahimi et al. [16] considered a hybrid flow shop problem in which each job is accompanied with an uncertain due date. The considered objective function is a combination of makespan and total tardiness; they proposed a number of metaheuristics to deal with this problem.

Ding et al. [15] considered a no-wait flow shop problem with the objective of minimizing the total tardiness; they proposed a heuristic that is designed to speed up the search by focusing on a subset of the jobs rather than all the jobs. Fernandez-Viagas and Framinan [17] studied a permutation flow show problem with a common due date for the jobs; they proposed two metaheuristics to deal with the problem and compared their results with the competitive methods. Shen et al. [54] developed a metaheuristic to deal with the no-idle permutation flow-shop scheduling problem with the objective of minimizing the total tardiness.

Perez-Gonzalez and Framinan [44] studied a permutation flow shop problem in which the jobs have a common due date. They considered two scenarios when a primary schedule is set up and new jobs arrive; first, to freeze the schedule and do not change it until all of the jobs are completed. Alternatively, to modify the schedule and accommodate the new jobs as long as the common due date is not violated. They performed computational experiments to determine which strategy works better when the jobs have certain characteristics.

Aldowaisan and Allahverdi [3] and later on, Aldowaisan and Allahverdi [2] considered the no-wait flow shop problem with due date constraints with the objective function of minimizing the total tardiness. They investigated the performance of various dispatching rules and introduced a simulated annealing and a genetic algorithm to deal with the problem.

Gupta and Kumar [24] developed a heuristic algorithm to minimize a combination of total tardiness and makespan. Samarghandi [50] studied the $F|nwt, d_j|C_{\max}$ and developed a particle swarm optimization to minimize the makespan; a Lagrangian relaxation method was proposed to deal with the violated due date constraints.

As one can notice, a common theme between all of the cited methods is that they first relax the due date constraints and then solve the remaining scheduling problem with a variant of the lateness/tardiness measure in the objective function by means of a heuristic or a metaheuristic algorithm.

2.2. Exact methods

Mathematical programming techniques have long been employed to solve sequencing and scheduling problems. Selen and Hott [53] developed a mixed integer programming for the flow shop problem. Stafford [55] developed a mixed integer linear programming (MILP) based on the all-integer model of Wagner [60].

Pekny and Miller [42] compared the performance of an exact algorithm with a number of heuristic and metaheuristic algorithms developed for $F|nwt|\gamma$. This algorithm was initially introduced by Miller and Pekny [36] to solve large-scale asymmetric travelling salesman problems. Later on, Pekny and Miller [43] proposed a