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Two phased hybrid local search for the periodic capacitated arc routing problem

Yuning Chen[a], Jin-Kao Hao[b,c,*]

[a] College of Information System and Management, National University of Defense Technology, Changsha 410073, PR China
[b] LERIA, Université d'Angers, 2 bd Lavoisier, Angers, Cedex 01 49045, France
[c] Institut Universitaire de France, 1 rue Descartes, Paris, Cedex 05 75231, France

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ABSTRACT

The periodic capacitated arc routing problem (PCARP) is a challenging general model with important applications. The PCARP has two hierarchical optimization objectives: a primary objective of minimizing the number of vehicles (F_v) and a secondary objective of minimizing the total cost (F_c). In this paper, we propose an effective two phased hybrid local search (HLS) algorithm for the PCARP. The first phase makes a particular effort to optimize the primary objective while the second phase seeks to further optimize both objectives by using the resulting number of vehicles of the first phase as an upper bound to prune the search space. For both phases, combined local search heuristics are devised to ensure an effective exploration of the search space. Experimental results on 63 benchmark instances demonstrate that HLS performs remarkably well both in terms of computational efficiency and solution quality. In particular, HLS discovers 44 improved best known values (new upper bounds) for the total cost objective F_c while attaining *all* the known optimal values regarding the objective of the number of vehicles F_v. To our knowledge, this is the first PCARP algorithm reaching such a performance. Key components of HLS are analyzed to better understand their contributions to the overall performance.

1. Introduction

Due to their theoretical hardness and practical importance, the periodic capacitated arc routing problem (PCARP) as well as many of its closely related problems in logistics have attracted considerable research effort in the last decades (Díaz-Madroñero, Peidro, & Mula, 2015). Compared to the popular capacitated arc routing problem (CARP) (Golden & Wong, 1981), the PCARP requires that the tasks are served for a certain number of times over a given multi-period horizon. The PCARP is typically encountered in waste collection applications, where we want to design a plan to collect the daily waste on each street in the city. In the PCARP that was first introduced in Lacomme, Prins, and Ramdane-Chérif (2002), streets may require several services for a multi-period time horizon (e.g., one week) according to a service pattern (e.g., a street requiring two services can be serviced by a Monday–Thursday or Tuesday–Friday pattern). The PCARP is to schedule vehicles to cover the required services of each day over the time horizon while optimizing two hierarchical objectives: a *primary* objective of minimizing the number of vehicles used over the time horizon (F_v) and a *secondary* objective of minimizing the total cost (F_c).

The PCARP is computationally challenging since it generalizes the classical and NP-hard CARP (Golden & Wong, 1981). Compared to its single-period special case – CARP – which has been intensively studied in the last decades (e.g., Beullens, Muyldermans, Cattrysse, & Oudheusden, 2003; Brandão & Eglese, 2008; Hertz, Laporte, & Mittaz, 2000; Martinelli, Poggi, & Subramanian, 2013; Mei, Tang, & Yao, 2009; Santos, Coutinho-Rodrigues, & Current, 2010; Tang, Mei, & Yao, 2009), the PCARP is somewhat less investigated. Due to its intrinsic intractability, existing research on the PCARP focuses mainly on designing effective heuristics to find high-quality near-optimal solutions in a reasonable time frame. As a first attempt to solve this problem, Chu, Labadi, and Prins (2004) presented several constructive heuristics. Later, two advanced heuristic algorithms were proposed: the Memetic Algorithm (LMA) by Lacomme, Prins, and Ramdane-Chérif (2005) and the Scatter Search algorithm (SS) by Chu, Labadi, and Prins (2006). Both approaches adapted the representation scheme and search operators of the classical CARP to the PCARP, and applied a greedy heuristic to build elite initial solutions of the population. Kansou and Yassine (2009) introduced an Ant colony heuristic with an

* Corresponding author at: LERIA, Université d'Angers, 2 bd Lavoisier, 49045 Angers, Cedex 01, France.
E-mail addresses: cynnudt@hotmail.com (Y. Chen), jin-kao.hao@univ-angers.fr, hao@info.univ-angers.fr (J.-K. Hao).

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efficient constructive procedure which outperformed the previous methods on a set of instances. Finally, Mei, Tang, and Yao (2011) presented another memetic algorithm with route-merging (MARM) which clearly dominated all previous PCARP approaches, making a significant improvement in PCARP solving. This approach will serve as the main reference for our algorithm assessment.

For a comprehensive literature review, we mention a close relative of the PCARP, called the Periodic Vehicle Routing Problem (PVRP), which is the vertex routing counterpart of the PCARP. The PVRP appeared earlier than the PCARP and consequently has received more research attention (e.g., (Cordeau, Gendreau, & Laporte, 1997; Drummond, Ochi, & Vianna, 2001; Francis & Smilowitz, 2006; Gaudioso & Paletta, 1992)). Different from the PCARP investigated in this work which is a bi-level optimization problem, the PVRP involves a single objective (the total cost objective). Moreover, the PVRP can be considered to be inherently less complex than the PCARP if we compare their single-period special cases (CARP vs. VRP). Indeed, a CARP with n tasks corresponds to a VRP with $2n+1$ vertices (Longo, de Aragão, & Uchoa, 2006). These observations also confirm the challenge of solving the PCARP compared to the PVRP.

In this work, we propose a two phased hybrid local search (HLS) approach for solving the PCARP with the following motivations and contributions.

- We notice that to handle the two hierarchical objectives, the existing studies on the PCARP typically optimize an aggregated weight function which is a linear combination of the two hierarchical objectives. Even if this approach is simple to implement, it does not explicitly recognize the priority of the primary objective and the algorithms using this approach need to explore a very large search space including many irrelevant solutions. Contrary to this objective aggregation approach, our HLS algorithm proposed in this paper relies on: (1) a first phase which focuses on the minimization of the number of vehicles, and (2) a second phase which uses the resulting number of vehicles as an upper bound to strongly constrain the optimization process.
- The proposed HLS algorithm integrates dedicated search operators and heuristics to ensure an effective search of both phases. To obtain an initial PCARP solution with a small number of vehicles, the first phase of HLS employs a specific tabu search procedure to *evenly* assign the tasks among the different periods of the time horizon and applies a heuristic CARP algorithm to generate vehicle routes for each period. In order to further ameliorate this initial PCARP solution, the second phase of HLS relies on two complementary local search procedures to reduce both the number of vehicles and the total cost. In particular, HLS uses the number of vehicles of the initial solution (from the first phase) as an upper bound to discard all candidate solutions whose number of vehicles is larger than the upper bound, and thus only explores a largely reduced search space.
- We assess our HLS algorithm on three sets of 63 popular benchmark instances in the literature. Our computational results indicate that HLS competes very favorably with the current best PCARP algorithms and is able to reach all the known optimal values in terms of F_v, and discovers 44 improved best values (new upper bounds) in terms of F_c which can be used to evaluate new PCARP algorithms. To our knowledge, no previous algorithm achieves such a performance.

The reminder of the paper is organized as follows. We introduce the PCARP in Section 2 and then present the HLS algorithm in Section 3. We show a performance assessment of the proposed algorithm and an analysis of the key elements of HLS in Sections 4 and 5 respectively, followed by concluding remarks in Section 6.

2. Problem description and solution representation

2.1. Problem description

Given a m-period time horizon $H = \{1, 2, \dots, m\}$ and an undirected graph $G(V, E)$ with vertex set V and edge set E, a set of required edges (also called tasks hereafter) E_R ($E_R \subset E$) and a fleet of identical vehicles with a capacity of Q that are based at the depot vertex v_d ($v_d \in V$). Let n be the number of required edges (i.e., $n = |E_R|$). Each edge $e = (i, j) \in E_R$ (a task), which is considered as a pair of arcs $< i, j >$ and $< j, i >$, is associated with a traversal cost ($tc(e)$). Additionally, each task $t \in E_R$ is associated with a service cost $sc(t)$, a service frequency $f(t)$ (based on which, an allowable service pattern set $ASP(t)$ is also given) and a demand vector $d(t) = \{d_1(t), d_2(t), \dots, d_m(t)\}$ where $d_x(t)$ ($x = 1, \dots, m$) indicates the intraperiod demand of period x of task t.

Let $ad(t, p, h)$ denote the accumulated demand of task t ($t \in E_R$) in period $h \in H$, where task t is served by pattern p ($p \in ASP(t)$). We recall that a pattern depicts the number of services provided for a task over a time horizon, and the periods when the service is performed. Once the pattern for a task is determined, its accumulated demand in a particular service period is calculated by summing up all intraperiod demands between last service period and the current one. For instance, a pattern $p_0 = \{2, 5\}$ is selected for task t_0 indicating t_0 is serviced on the second day (i.e., Tuesday) and fifth day (i.e., Friday) of the week, then the accumulated demand of task t_0 on Tuesday is $ad(t_0, p_0, 2) = d_5(t_0) + d_6(t_0) + d_7(t_0) + d_1(t_0)$ and on Friday is $ad(t_0, p_0, 5) = d_2(t_0) + d_3(t_0) + d_4(t_0)$.

The PCARP amounts to deciding a pattern p ($p \in ASP(t)$) for each task t ($t \in E_R$) and to designing a set of vehicle routes for each period h ($h \in H$), with the purpose of minimizing the number of vehicles (F_v) used over the time horizon H as the primary objective, and minimizing the total cost of all vehicle routes (F_c) as the second objective, while respecting the following constraints: (1) each vehicle route starts and ends at the depot v_d; (2) each task t ($t \in E_R$) is served no more than once in each period h ($h \in H$); (3) the service pattern selected for each task t ($t \in E_R$) must be from its allowable service pattern set $ASP(t)$; (4) the total demand serviced on the route of a vehicle must not exceed the vehicle capacity Q. A mathematical formulation of the PCARPA was described in Mei et al. (2011), which is based on a solution representation different from the representation we adopted (described below).

2.2. Solution representation

Our HLS algorithm uses the following solution representation to encode the candidate solutions of the PCARP. First, each task (i.e., each required edge) is assigned two IDs $(a, a+n)$ $(a = 1, \dots, n)$ to represent the two associated arcs of the task. We also define a dummy task with 0 as its task ID and both its head and tail vertices being the depot vertex v_d. This dummy task is to be inserted somewhere in the solution as a route delimiter. A candidate solution S of the PCARP is then represented by m (number of periods) CARP solutions, i.e., $S = \{S_1, S_2, \dots, S_m\}$. Suppose each CARP solution S_i ($i \in \{1, \dots, m\}$) involves n_i tasks and r_i vehicle routes, S_i can then be encoded as an order list of $(n_i + r_i + 1)$ task IDs among which $(r_i + 1)$ are dummy tasks: $S_i = \{S_i(1), S_i(2), \dots, S_i(n_i + r_i + 1)\}$, where $S_i(j)$ is a task ID (an arc of the task or a dummy task) in the jth position of S_i. S_i can also be written as a set of r_i routes: $S_i = \{0, R_{i1}, 0, R_{i2}, 0, \dots, 0, R_{ir_i}, 0\}$, where R_{ij} denotes the jth route composed of $|R_{ij}|$ task IDs (arcs), i.e., $R_{ij} = \{R_{ij}(1), R_{ij}(2), \dots, R_{ij}(|R_{ij}|)\}$, with $R_{ij}(k)$ being the task ID at the kth position of R_{ij}. Let $dist(u, v)$ denote the shortest path distance between the head vertex of arc u ($head(u)$) and the tail vertex of arc v ($tail(v)$). The primary objective value $F_v(S)$ and the secondary objective value $F_c(S)$ of the candidate solution S can be