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Innovative Applications of O.R.

An integrated assortment and shelf-space optimization model with demand substitution and space-elasticity effects

Alexander Hübner*, Kai Schaal

Department of Operations, Catholic University Eichstätt-Ingolstadt, Auf der Schanz 49, Ingolstadt 85049, Germany

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Retailers must define their assortments and assign shelf space to the items included in these assortments. These two planning problems are mutually dependent if space is scarce. We formulate a model that maximizes a retailer's profit by selecting the optimal assortment and assigning limited shelf space to items. This model is the first decision model to integrate assortment and shelf-space planning by considering stochastic and space-elastic demand, out-of-assortment and out-of-stock substitution effects. To solve the model, we develop a specialized heuristic that efficiently yields near-optimal results, even for large-scale problems. We show that our approach outperforms alternative approaches, e.g. a sequential planning approach that first picks assortments and then assigns shelf space by up to 18%, and a greedy algorithm by up to 16% in terms of profit.

We test our model on two real data sets for perishable and non-perishable items and show how it can support retailers in increasing their profits by up to 25%. We then use the model to generalize these results and find that space elasticity and substitution effects have a significant impact on profits, assortment size as well as facing decisions, and that both effects reinforce each other. Using our model, we finally derive rules-of-thumb for planners in practice.

1. Introduction

In this paper, we develop and solve a version of the joint assortment and shelf-planning problem. We propose a model that helps retailers determine the variety of items to offer and the shelf space these items are assigned. These two decision problems are interdependent when shelf space is limited. Offering broader assortments with more products limits the space available per product and vice versa. Consequently, planning retail shelves makes it necessary to specify the products to be carried on each shelf and determine the space and quantity to be assigned to selected items. These decisions not only depend on the margins of the products, but also on customer preferences and the associated demand. If products are not available, customers may settle for another similar product instead, i.e. they substitute items. Furthermore, the more shelf space is allocated to an item, the more the item becomes visible to customers and the more demand for it increases. This is referred to as "space-elastic demand."

This assortment- and shelf-planning problem is of great importance to retailers because the increasing number of products conflicts with limited shelf space. Nowadays, up to 30% more products compete for the scare space than was the case ten years ago (EHI, 2014). Availability of the right products is one of the main drivers behind customer satisfaction. Retailers have recognized this fact and identified assortment optimization as a key task for achieving superior performance (Eltze, Goergens, & Loury, 2013) and improving space productivity (Gutgeld, Sauer, & Wachinger, 2009). In fact, shelf space has been referred to as the retailer's scarcest resource (cf. e.g. Brown & Tucker, 1961; Geismar, Dawande, Murthi, & Sriskandarajah, 2015; Irion, Lu, Al-Khayyal, & Tsao, 2012; Lim, Rodrigues, & Zhang, 2004). The increasing number of items to allocate, the shortage of shelf space, narrow margins in retail, and the intensity of competition have greatly magnified the importance of retail assortment and shelf-space planning (cf. Hübner, Kuhn, & Sternbeck, 2013).

The topic of integrated assortment and shelf planning is also relevant from a research perspective. Despite the mutual dependence of the two decisions, current literature addresses the assortment and shelf-space planning problems separately (cf. Bianchi-Aguiar, Hübner, Carravilla, & Oliveira, 2016; Hübner & Kuhn, 2012; Kök, Fisher, & Vaidyanathan, 2015). Assortment models largely deal with substitution effects, but do not consider space-elastic demand, and furthermore often disregard limited shelf space. Shelf planning models mainly focus on space-elastic demand for a given

* Corresponding author.
E-mail addresses: alexander.huebner@ku.de (A. Hübner), kai.schaal@ku.de (K. Schaal).

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assortment and thus do not account for substitution in cases where customers intend to replace delisted items with substitute items. Furthermore, the stochastic nature of demand is often ignored and demand assumed to be deterministic.

In this paper, we model and solve the problem of determining the assortment and allocating quantities for each product to a retail shelf. The model accounts for stochastic demand, substitution and space-elasticity effects. We build on a constraint multi-item newsvendor formulation and a set of substitutable products.

The remainder is organized as follows: Section 2 further specifies the setting; Section 3 discusses the literature relevant to our problem. Section 4 formulates the model and presents a solution algorithm. Section 5 tests the performance of our algorithm, applies the model to two case studies and then generalizes these results. Section 6 draws conclusions and presents an outlook on future research topics.

2. Setting and planning problem

Decision problem. The number of products carried in a retail store can be very large. In the grocery industry, supermarkets often carry up to 60,000 items (cf. EHI, 2014). These are organized in merchandising categories, such as beverages, confectionary or canned food. Categories can be further divided into subcategories, so that the difference between products within a subcategory is minimal, but the difference between subcategories is significant. For example, subcategories in the confectionary category include chocolate, chips, candy, etc. Because assortment and shelf plans are made for each subcategory, we consider each subcategory separately and assume that product demand depends on the decisions within a subcategory. A subcategory typically contains up to 400 items, but usually around 60–80 items on average. The shelf space available for a subcategory is limited and determined by preceding decisions regarding store layout planning (cf. Hübner et al., 2013).

The retail shelf planner in charge of a subcategory selects the products to be included in the assortment. This is referred to as the "listing decision." The listing decision is closely connected to a second decision, referred to as the "facing decision," which determines how many facings a listed item is assigned. A facing is the first visible unit of a particular product in the front row of a shelf. If shelf space is limited, the two decisions are mutually dependent, because, for instance, the listing of additional items requires a reduction in facings or the delisting of other selected items. In the literature, the facing decision is sometimes referred to as the "shelf layout decision" or "shelf-space allocation." Because we explicitly refer to the determination of the number of facings for listed items, we use the term "facing decision" below. Note that the listing decision can be integrated into the facing decision by allowing zero facings, which corresponds to a delisting.

Changes in the facing decision for an item imply changes in the available total shelf quantity of this item. Behind each facing of an item i (k_i), there is more available shelf space which retailers fill up with further units of the respective item. This is called the stock per facing (g_i). The stock a retailer can place behind a facing depends on the shelf depth and the physical size of an item unit. The product of the number of facings and the stock per facing determines the total shelf quantity ($x_i = k_i \cdot g_i$) of an item that is available for demand fulfillment (cf. Fig. 1).

The retailer seeks to offer those item shelf quantities that match customer demand. Offering a shelf quantity larger than customer demand results in excess inventory and costs for disposing of perishable products at the end of their shelf life or inventory carrying costs in the case of non-perishable products (cf. Kök & Fisher, 2007). Offering quantities lower than demand results in unfilled demand and lost sales if customers cannot find the items they intend to purchase or if they do not find an appropriate substitute. In summary, the retailer needs to determine assortments (i.e. make listing decisions) and decide how many facings per selected item to allocate to the shelf (i.e. the facing decision). The retailer ultimately determines the total shelf quantities for listed items that are used to satisfy customer demand and aims to maximize the expected total profit across all listed items. Total profit consists of the expected total margins, less the cost of unfulfilled demand and excess inventory.

Related demand effects. The listing and facing decisions impact customer demand in three ways:

(1) Space-elastic demand. Customer demand for an item is space-elastic, i.e. it increases with an increasing number of facings assigned to a respective item. For example, Brown and Tucker (1961), Frank and Massy (1970), Curhan (1972) and Drèze, Hoch, and Purk (1994) conducted various experiments and analyzed the magnitude of space-elasticity effects. Chandon, Hutchinson, Bradlow, and Young (2009) show that the variation of facings is the most significant in-store factor, even stronger than positioning and pricing. Desmet and Renaudin (1998) state that increasing impulse buying rates of an item also increase its space elasticity. Using a meta-analysis, Eisend (2014) identifies an average demand increase of 17% every time the number of facings is doubled. With regard to cross-space elasticities, i.e. the fact that demand for one item changes due to a change in the number of facings of other items, Brown and Lee (1996) and Kök et al. (2015) state that there is no empirical evidence that product-level demand can be modeled with cross-space elasticities. Zufryden (1986) finds that at an individual level, considering cross-space elasticities would result in an enormous number of required estimations. Eisend (2014) calculates an average cross-space elasticity of −1.6% and Hübner and Schaal (2016b) find that facing decisions are only significantly affected if cross-space elasticity differs significantly from this value. We therefore disregard cross-space elasticities in the following. Mutual dependencies between the products arise from customer substitutions.

(2) Out-of-assortment and (3) Out-of-stock substitution demand. Customers can substitute for their choice if items are unavailable. For example, Gruen, Corsten, and Bharadwaj (2002), Aastrup and Kotzab (2009) and Tan and Karabati (2013) show that between 45% and 84% of the demand can be substituted. Unavailability of items can result from two cases: Either an item is delisted as a consequence of the assortment decision (out-of-assortment, OOA) or it is temporarily unavailable and currently not available on the shelf (out-of-stock, OOS). In both situations, customers might replace the unavailable items with other items, which results in demand increases for the respective substitutes.

Table 1 summarizes the trade-offs between assortment and shelf-space decisions, and describes the impact on demand and demand fulfillment.

To account for these interdependencies between the planning problems and the relevant demand effects, an integrated model that simultaneously optimizes assortments and shelf-space assignment is required. In the next section, we investigate the literature on existing optimization approaches relevant to the problem introduced in this paper.

3. Related literature and contribution

We focus on literature which support retailers in solving the trade-offs described in the previous section. Many existing contributions treat the two topics of assortment and shelf planning largely separately and propose approaches to support retailers in either one decision or the other. Therefore, we review the literature