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Discrete Optimization

Solving the maximum min-sum dispersion by alternating formulations of two different problems

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ABSTRACT

The maximum min-sum dispersion problem aims to maximize the minimum accumulative dispersion among the chosen elements. It is known to be strongly NP-hard problem. In this paper we present heuristic where the objective functions of two different problems are shifted within variable neighborhood search framework. Though this heuristic can be seen as an extended variant of variable formulation search approach that takes into account alternative formulations of one problem, the important difference is that it allows using alternative formulations of more than one optimization problem. Here we use one alternative formulation that is of max-sum type of the originally max–min type maximum diversity problem. Computational experiments on the benchmark instances used in the literature show that the suggested approach improves the best known results for most instances in a shorter computing time.

1. Introduction

Given a set N of $n = |N|$ elements labeled by integer numbers from 1 to n, and the distance d_{ij} between any two elements i and j, a dispersion or diversity problem (DP) consists generally of finding a subset $S \subset N$ so that an objective function based on the distances between elements in S is maximized or minimized. According to the objective function, two main classes of dispersion problems are distinguished: those that use efficiency-based objective functions and those that use equity-based objective functions. An efficiency-based objective function reflects the dispersion quantity for the entire selection S, while an equity-based objective function guarantees equitable dispersion among the selected elements. Among widely studied problems that use efficiency-based objective functions are: the maximum diversity problem (MDP), whose goal is to find a subset S so that the sum of the distances between the selected elements is maximized, and the max–min diversity problem (MMDP), where the goal is to find a subset S so that the minimum distance between the selected elements is maximized. On the other hand, the problems that consider equity-based measures introduced by Prokopyev, Kong, and Martinez-Torres (2009) are: the maximum mean dispersion problem (max-mean DP), the minimum differential dispersion problem (min-diff DP), and the maximum min-sum dispersion problem (max–min-sum DP). The first of these problems requires finding a subset S, so that the average distance between the selected elements is maximized, while the second one concerns finding a subset S so that the difference between the maximum sum and the minimum sum of the distances from a node in S to the other selected elements in S is minimized. Finally, the max–min-sum DP consists in finding a subset S so that the minimum sum of the distances to the other selected elements is maximized. Except max-mean DP, the cardinality of the subset S must be equal to a given number m.

Diversity problems that use efficiency-based measures find their application in the context of facility location (locating facilities according to distance, accessibility, impacts, etc.) (Erkut, 1990; Erkut & Neuman, 1989; Kuby, 1987; Rahman & Kuby, 1996), maximally diverse/similar group selection (e.g., biological diversity, admissions policy formulation, committee formation, curriculum design, market planning, etc.) (Adil & Ghosh, 2005; Ghosh, 1996; Glover, Kuo, & Dhir, 1998; Kuo, Glover, & Dhir, 1993; Weitz & Lakshminarayanan, 1998), and densest subgraph identification (Kortsarz & Peleg, 1993). On the other hand, diversity problems that use equity-based measures have applications in the context of urban public facility location, where the fairness among candidate facility locations is important (Teitz, 1968), selection of homogeneous groups (Brown, 1979a), dense/regular subgraph

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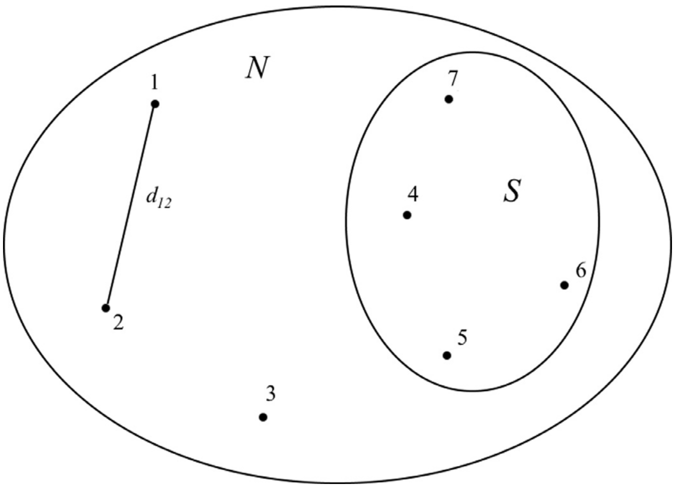

Fig. 1. Arbitrary selected subset S with cardinality $m = 4$.

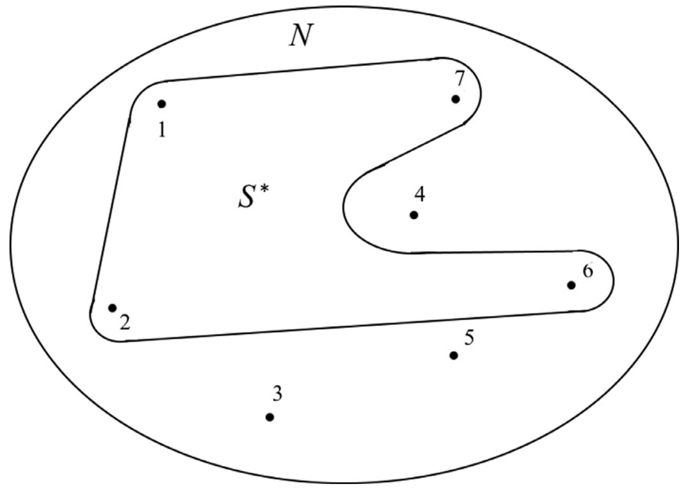

Fig. 2. Optimal solution $S^* = \{1, 2, 6, 7\}$ with $f(S^*) = 30$.

identification (Kortsarz & Peleg, 1993), and equity-based measures in network flow problems (Brown, 1979b).

In this paper, we study the maximum min-sum dispersion problem (max–min-sum DP) aimed to select a set $S \subset N$ containing m elements so that the smallest total dispersion associated with each selected element i is maximized. Let S be a subset of N with cardinality m, then the total dispersion associated with each selected element $i \in S$ is given as $\Delta(i) = \sum_{j \in S, j \neq i} d_{ij}$.

Example. For simplicity of understanding, let us illustrate max–min-sum DP and the notation which we will use on the example involving the set N with $n = 7$ elements; the distance matrix D given in Eq. (1); and the cardinality $m = 4$ of the sought subset S.

$$D = \begin{bmatrix} - & 8 & 11 & 7 & 11 & 13 & 9 \\ 8 & - & 6 & 9 & 9 & 13 & 14 \\ 11 & 6 & - & 7 & 5 & 10 & 12 \\ 7 & 9 & 7 & - & 4 & 6 & 5 \\ 11 & 9 & 5 & 4 & - & 4 & 8 \\ 13 & 13 & 10 & 6 & 4 & - & 7 \\ 9 & 14 & 12 & 5 & 8 & 7 & - \end{bmatrix} \tag{1}$$

Since 4 out of 7 elements should be chosen, the solution space contains $\binom{7}{4} = 35$ possible subsets. The subset $S = \{4, 5, 6, 7\}$, presented in Fig. 1, is chosen arbitrary as one of the candidate solutions. The objective function value of this solution equals to $f(S) = 15$ and is calculated as:

$$\begin{aligned} f(S) &= \min\{\Delta(4), \Delta(5), \Delta(6), \Delta(7)\} \\ &= \min\{d_{45} + d_{46} + d_{47}, d_{54} + d_{56} + d_{57}, d_{64} \\ &\quad + d_{65} + d_{67}, d_{74} + d_{75} + d_{76}\} \\ &= \min\{15, 16, 17, 20\} = 15. \end{aligned}$$

However, by the complete enumeration of all 35 solutions, we found that the optimal solution for the considered example is $S^* = \{1, 2, 6, 7\}$. This optimal solution is presented in Fig. 2 and has the objective function value $f(S^*) = 30$.

Mathematical formulation. Using the binary variable x_i to indicate if the element i is selected or not, the max–min-sum DP may be formulated as follows (Prokopyev et al., 2009):

$$\max \left\{ \min_{i \in N} \sum_{j \in N, j \neq i} d_{ij} x_i x_j \right\} \tag{2}$$

subject to

$$\sum_{i \in N} x_i = m \tag{3}$$

$$x \in \{0, 1\}^n \tag{4}$$

The above formulation may be linearized yielding the following 0–1 mixed integer program (Prokopyev et al., 2009):

$$\max\ r \tag{5}$$

subject to

$$r \leq \sum_{j \in N} d_{ij} x_j + Q(1 - x_i),\ i \in N \tag{6}$$

$$\sum_{i \in N} x_i = m \tag{7}$$

$$r \in \mathbb{R},\ x \in \{0, 1\}^n \tag{8}$$

where Q represents the “big-M” coefficient. Note that the value of Q may be set as (Prokopyev et al., 2009):

$$1 + \max_{i \in N} \sum_{j \in N} \max(d_{ij}, 0) - \sum_{j \in N} \min(d_{ij}, 0).$$

The continuous variable r represents the value of the objective function. It is bounded from above by constraint (6). In fact, if the element i is selected, the right-hand side of the constraint (6) associated to the element i equals to the total dispersion associated to it, otherwise it becomes much larger than all total dispersions, and the constraint is relaxed. Thus, constraint (6) guarantees that the maximum value of r equals to the minimum of the total dispersions of the elements belonging to the solution. Constraint (7) has the same meaning as in the previous formulation and guarantees that the solution has required cardinality m.

The max–min-sum DP is a strongly NP-hard problem (Prokopyev et al., 2009). For solving it, several approaches have been proposed in the literature. Prokopyev et al. (2009) used CPLEX 9.0 MIP solver to solve the above MIP formulation, and they also proposed a generic GRASP heuristic for solving dispersion problems using equity-based measure.

More recently, (Aringhieri, Cordone, & Grosso, 2015) proposed several construction procedures and a Tabu search based heuristic for max–min-sum DP. In addition, they introduced a new set of test instances for max–min-sum DP. The Tabu search of Aringhieri et al. (2015) may be considered as a state-of-the-art heuristic for solving max–min-sum DP.

In Martínez-Gavara, Campos, Laguna, and Martí (2016), the authors proposed several GRASP based heuristics that incorporate the strategic oscillation paradigm within the construction phase of the GRASP framework. In addition, they developed a simple Tabu

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