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Priority degrees for hesitant fuzzy sets: Application to multiple attribute decision making

Jibin Lan, Ruifang Jin*, Zhaoyi Zheng, Mingming Hu

College of Mathematics and Information Science, Guangxi University, Nanning, Guangxi 530004, PR China

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A B S T R A C T

In this paper, some drawbacks to existing ordering relations for hesitant fuzzy sets are examined using examples. To overcome these flaws, a priority degree formula for comparing two hesitant fuzzy sets is presented and the desirable priority degree properties studied. Then, based on an introduced priority degree formula, a new hesitant fuzzy multiple attribute decision making methodology is proposed. Finally, a numerical example together with a comparison analysis is given to illustrate the effectiveness and feasibility of the new approach to decision making applications.

1. Introduction

The fuzzy sets theory introduced by Zadeh [1] has been very successful in dealing with problems involving uncertainty. With an increase in inaccurate and vague information in real life problems, several extensions of the fuzzy set have been developed, one of which is the intuitionistic fuzzy set (IFS) pioneered by Atanassov [2], which has a membership function, a non-membership function and a hesitancy function. Zadeh [3] presented a type-2 fuzzy set that allowed the membership of a given element to be a fuzzy set. The type-n fuzzy set [4] generalized type-2 fuzzy set, thereby permitting the membership to be a type-n-1 fuzzy set. The fuzzy multiset introduced by Yager [5] allowed elements to be repeated more than once.

In practical applications, because of a lack of knowledge, time pressure and other reasons, people do not often agree on specific elements in complex decisions, which means that it is often difficult to reach agreement. For example, two decision makers may discuss the membership degree of an element x to a set A, for which one decision maker wishes to assign 0.4 but the other wishes to assign 0.8. Accordingly, the difficulty in establishing a common membership degree is not because there is a margin of error or some possibility distribution values, but because there is a set of possible values [6]. To deal with such cases, Torra [7] and Torra and Narukawa [8] proposed the concept of the hesitant fuzzy set(HFS), which permitted membership to have a set of possible values.

* Corresponding author.
E-mail addresses: lanjibin@gxu.edu.cn (J. Lan), jinruifang999@163.com (R. Jin).

Since the concept of the hesitant fuzzy set was established, it has gained increasing attention [9–17] and has been successfully applied to many uncertain decision making problems. Many studies have also been conducted on the application of HFS aggregation operators [18–20] and distance and similarity measures [21–23] to multi-criteria decision making problems.

Multi-criteria decision making has been widely applied in many scientific fields [24–26], such as medical care [27], engineering [28], social sciences [29] and economics [30]. In general, multiple attribute decision making problems have two phases; aggregation and exploitation. Of these, the aggregation phase is more important, so significant aggregation techniques have been developed for decision-making processes, in which the experts express their assessments using HFSs. Xia and Xu [31] presented a hesitant fuzzy weighted averaging operator and a hesitant fuzzy weighted geometric operator, to which they gave different extensions and generalizations; a generalized hesitant fuzzy weighted averaging operator, a generalized hesitant fuzzy weighted geometric operator, a hesitant fuzzy hybrid averaging operator, and a hesitant fuzzy hybrid geometric operator. Yu et al. [32] proposed a new hesitant fuzzy aggregation operator based on the Choquet integral which included the importance of the elements, their ordered positions and a fuzzy measure. Motivated by the idea of prioritized aggregation operators, Wei [33] proposed a hesitant fuzzy prioritized weighted average and hesitant fuzzy prioritized weighted geometric aggregation operators, which accounted for the different criteria priority levels in multi-criteria decision-making problems. Yu and Zhou [34] defined a generalized hesitant fuzzy Bonferroni mean which extended the Bonferroni mean to a hesitant fuzzy environment. Other extensions of the Bonferroni mean were

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proposed in [35,36]. Xia et al. [37] introduced a new HFS operator by extending the quasi-arithmetic means. Bedregal et al. [38] presented two methodologies to develop triangular hesitant aggregation functions over all THFS. Xu and Xia [39] proposed several distance and similarity measures and studied the properties and relationships between them. Zhou and Li [40] modified the axiom definitions for the distance and similarity measures developed by Xu and Xia [39], and proposed some new distance and similarity measures between HFSs based on Hamming and Euclidean distances. Peng et al. [41] presented a novel generalized hesitant fuzzy synergetic weighted distance measure which reflected both individual distances and their ordered positions. In addition to these, many approaches can be used to deal with multiple attribute decision making problems(see, [42–47]). An exploitation phase was developed to build the preference relations between the alternatives and a nondominant choice degree was applied to obtain a solution set of alternatives for multiple attribute decision making problems.

Ordering relations play an important role in decision making and some HFS ordering relations have been proposed. Rodriguez et al. [48] gave a definition for order relations between HFSs, and then used aggregation operators to determine the order relations between them. Xia and Xu [14] introduced a comparison law by defining a score function to determine the order relations between HFSs. Farhadinia [49] also developed two ordering methods for HFSs. Zhou [50] introduces the intuitionistic fuzzy ordered weighted cosine similarity (IFOWCS) measure by using the cosine similarity measure of intuitionistic fuzzy sets and the generalized ordered weighted averaging (GOWA) operator. Zhou [51] develops the continuous intuitionistic fuzzy ordered weighted distance (C-IFOWD) measure by using the continuous intuitionistic fuzzy ordered weighted averaging (C-IFOWA) operator in the interval distance. Wei et al. [52] define the Shapley value-based L_p-metric and extend VIKOR method with the L_p-metric to deal with the correlative multiple criteria decision making (MCDM) problem under hesitant fuzzy environment. Wei et al. [53] developed hesitant fuzzy choquet ordered averaging (HFCOA) operator and hesitant fuzzy choquet ordered geometric (HFCOG) operator, and apply the HFCOA and HFCOG operators to hesitant fuzzy multiple attribute decision making. Zhao et al. [54] utilize Einstein operations to develop hesitant fuzzy Einstein correlated averaging (HFECA) operator and hesitant fuzzy Einstein correlated geometric (HFECG) operator. They can not only consider the importance of the elements or their ordered positions, but also reflect the correlation among the elements or their ordered positions. In addition to these, many aggregation operators and methods can be used to rank HFSs (see, [18,55–58]). However, the existing order relations for HFSs are defective(see examples2–4). For example, Jack and Tom play a game that has three turns. Jack has three cards; a 9 of spades, a 6 of spades and a 3 of spades; and Tom has three cards; an 8 of spades, a 5 of spades and a 2 of spades. The rules of the game are: (1)Each person can only select one of their own cards to play in each turn; (2)The card with the higher points wins in each turn; (3)The person who wins two turns is the final winner. Although Jack's cards have a points' advantage, it is not certain that Jack can win the game, as his winning probability is 0.6667. From the example above, we can construct two HFEs; $H_1(x) = \{0.9, 0.6, 0.3\}$ and $H_2(x) = \{0.8, 0.5, 0.2\}$. From the existing HFS order relations, we have $H_1(x) \succeq H_2(x)$, which does not conform to the actual situation.

This paper seeks to overcome the flaws outlined above. The remainder of this paper is organized as follows: Section 2 reviews some basic concepts and order relations. Section 3 analyzes the order relations between HFSs, and in Section 4, a priority degree formula for comparing two hesitant fuzzy sets is presented and the desirable priority degree properties studied. In Section 5, based on the proposed formula for the priority degree, a new approach to hesitant fuzzy multiple attribute decision making is developed. Section 6 gives an example to illustrate the rationality and applicability of the new method and in Section 7 conclusions are given.

2. Preliminaries

In this section, the HFS concept and order relations are briefly reviewed.

2.1. Hesitant fuzzy sets

As people are usually hesitant when making decisions, it is often difficult to reach a final agreement. With these difficulties in mind, Torra [59] developed the following hesitant fuzzy set definition:

Definition 1. [59] Given a fixed set X, then a hesitant fuzzy set (HFS) on X is a function that when applied to X returns a subset of values in [0,1].

For convenience, Wei [33] completed the original HFS definition by including the HFS mathematical representation as follows:

$$E = (\langle x, h_E(x)\rangle | x \in X).$$

where $h_E(x)$ is a set of some values in [0,1], and denotes the possible membership degree of the element $x \in X$ to the set E; $h(x) = h_E(x)$ is called a hesitant fuzzy element (HFE).

Example 1. Suppose that $X = \{x_1, x_2, x_3\}$ is the discourse set, and $h_M(x_1) = \{0.8, 0.5, 0.3\}, h_M(x_2) = \{0.6, 0.4\}$ and $h_M(x_3) = \{0.6, 0.3, 0.2\}$ are the HFEs for $x_i (i = 1, 2, 3)$ to a set E. Then E can be considered a HFS; i.e.

$E = \{\langle x_1, \{0.8, 0.5, 0.3\}\rangle, \langle x_2, \{0.6, 0.4\}\rangle, \langle x_3, \{0.6, 0.3, 0.2\}\rangle\}$.

Further operations on HFEs can be seen in [7,14].

2.2. Order relations between HFSs

Order relations play an important role in decision making. We first review some order relations.

Definition 2. [60] Given two HFSs, H_1 and H_2 on X of the same cardinality, it is defined that $H_1 \geq H_2$ if $H_1(x) \geq H_2(x)$ for all x. Note that $H_1(x)$ and $H_2(x)$ are HFEs. Here, $h_1 \geq h_2$ for HFEs h_1 and h_2 if $h_1^{\sigma(j)} \geq h_2^{\sigma(j)}$ for all $j = \{1, \cdots, |H_1|\}$, where $h^{\sigma(j)}$ is the jth element in h when they are ordered in a decreasing order.

Definition 3. [60] Let φ be a function on the HFSs such that the cardinality of φ is the same for all HFSs. We then say that φ is monotonic when $\varphi(E) \geq \varphi(E')$ for all $E = \{H_1, \cdots, H_n\}$ and $E' = \{H'_1, \cdots, H'_n\}$ such that $H'_i \geq H_i$ for all $i = \{1, \cdots, n\}$.

Definition 4. [60] Let $E = \{H_1, \cdots, H_n\}$ be a set of n HFSs and Θ a function, Θ: $[0, 1]^n \to [0, 1]$, we then export the Θ on the fuzzy sets to HFSs, which is defined as:

$$\Theta_E = \cup_{\gamma \in H_1(x) \times \cdots \times H_n(x)} \{\Theta(\gamma)\} \qquad (1)$$

Proposition 1.[60] Let $E = \{H_1, \cdots, H_n\}$ and $E' = \{H'_1, \cdots, H'_n\}$ such that $H'_i \geq H_i$ for all $i = \{1, \cdots, n\}$. Then, if Θ is a monotonic function, Θ_E is monotonic.

In practical applications, situations arise in which the number of the elements in the different hesitant fuzzy elements may vary. For correct operations, Xu and Xia [6] proposed the following regulation: the shorter element is extended by adding the minimum value, the maximum value, or any value until it has the same length as the longer element. The selection of this value depends mainly on the decision makers' risk preferences. Optimists expecting desirable outcomes may add the maximum value, while pessimists expecting unfavorable outcomes may add the minimum value.