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Composite adaptive locally weighted learning control for multi-constraint nonlinear systems

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ABSTRACT

A composite adaptive locally weighted learning (LWL) control approach is proposed for a class of uncertain nonlinear systems with system constraints, including state constraints and asymmetric control saturation in this paper. The system constraints are tackled by considering the control input as an extended state variable and introducing barrier Lyapunov functions (BLFs) into the backstepping procedure. The system uncertainty is approximated by a composite adaptive LWL neural networks (NNs), in which a prediction error is constructed by using a series-parallel identification model, and NN weights are updated by both the tracking error and the prediction error. The update law with composite error feedback improves uncertainty approximation accuracy and trajectory tracking accuracy. The feasibility and effectiveness of the proposed approach have been demonstrated by formal proof and simulation results.

1. Introduction

State and control constraints exist in many mechanical systems and industrial processes due to safety and performance consideration. Almost all real-world systems have nonlinear dynamics and model uncertainties. Control of uncertain nonlinear constrained systems is challenging and has gained more and more attention [1–4].

Control approaches for constrained systems include model predictive control (MPC) [5–7], reference governors (RGs) [8], and barrier Lyapunov functions (BLFs) [9–15]. In MPC, system constraints are explicitly considered and the control law is obtained by solving online receding horizon optimizations. In RGs, system constraints are guaranteed by the redesign of reference signals obtained by solving online optimizations. MPC and RGs have been considered as effective ways to tackle state constraints. However, high computational complexity and the requirement on high system modeling accuracy bring difficulties in applications of MPC and RGs to real-time control of uncertain nonlinear systems. Recently, barrier Lyapunov function (BLF)-based control for constrained nonlinear systems has gained more and more attention [9–13]. The function values of BLFs will grow to infinity if the arguments approach the constraints boundary resulting in constraints violation. The avoidance of constraints violation can be reached by bounding the BLFs [9]. BLF-based controllers have been designed for the nonlinear systems with time-invariant output constraints [9–11], time-varying output constraints [12], and full state constraints [13].

Neural network (NN) control has been widely developed for uncertain nonlinear systems due to the inherent approximation abilities of NNs [14–16]. The newly developed locally weighted learning (LWL) NNs apply independently adjusted local models to approximate nonlinear uncertainties [17–19]. The advantages of LWL approximation include [20]: (1) Easy learning from the continuous stream of training data in real time; (2) negative interference avoidance for their abilities in retaining all training data; (3) allowance of quick identification due to simple learning rules with a single optimum for building a local model. Conventional adaptive NN control is directed towards achieving stability of the closed-loop system by updating NN weights via only tracking errors. By updating NN weights through both prediction errors and tracking errors, composite adaptive control has been proposed for uncertain nonlinear systems to improve both identification accuracy and tracking accuracy [21–30].

This paper considers the adaptive control design for a class of high-order uncertain nonlinear systems with system constraints,

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including state constraints and asymmetric control saturation. The system constraints are tackled by considering the control input as an extended state and introducing symmetric/asymmetric BLFs into the backstepping procedure. Computational complexity of the backstepping design is significantly decreased by using a dynamic surface control technique [31,32]. The system uncertainty is approximated by a composite adaptive LWL NN, where the prediction error is constructed by using a serial-parallel estimation model through the design of a NN state observer. Compared with existing works, the main contributions of this study include:

1. By considering the control input as an extended state and introducing BLFs, the state constraints and the asymmetric control saturation are tackled, which extends current research on BLF-based control for nonlinear systems with state/output constraints to state constraints and asymmetric control saturation.
2. By constructing a serial-parallel estimation model and feeding the prediction error back the update law, the composite adaptive LWL NN is designed to approximate the system uncertainty, which improves both approximation accuracy and tracking accuracy.

2. Problem construction and preliminaries

2.1. Problem Formulation

Consider the following nth order SISO nonlinear system:

$$\dot{x}_i = x_{i+1},\ \ i = 1, 2, \ldots, n-1 \tag{1}$$

$$\dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u \tag{2}$$

where $x_i \in R$ and $u \in R$ are the state variable and the control input, respectively, and $f(\mathbf{x})$ and $g(\mathbf{x})$ are unknown nonlinear functions with $\mathbf{x} = [x_1, \ldots, x_n]^T$. The system (1) and (2) is constrained by the state constraint and control saturation:

$$|x_i| \le k_{c,i},\ -u_{c1} \le u \le u_{c2},\ \ i = 1, 2, \ldots, n \tag{3}$$

where $k_{c,i}$, u_{c1} and u_{c2} are known positive constants. We call the control input constraint as symmetric control saturation if $u_{c1} = u_{c2}$ and asymmetric control saturation if $u_{c1} \neq u_{c2}$. The objective of this paper is to design a BLF-based LWL NN control u such that $x_1(t)$ tracks a desired trajectory $y_d(t) \in R$ without violation of the system constraints described by (3).

Assumption 1. $f(\mathbf{x})$ is locally Lipschits continuous.

Assumption 2. $g(\mathbf{x})$ is locally Lipschits continuous and $0 < g_0 \le g(\mathbf{x}) \le g_1$ for $\forall \mathbf{x} \in D \triangleq \{\mathbf{x} \in R^n : |x_i| \le k_{c,i},\ i = 1,\ 2,\ \ldots,\ n\}$, where $g_0 > 0$, $g_1 > 0$ are positive constants with g_0 being known.

Assumption 3. $y_d(t)$ and the jth-order time derivatives $y_d^{(j)}(t)$, $j = 1, 2, \ldots, n$ are known and satisfy $|y_d| \le A_0 < k_{c1}$ and $|y_d^{(j)}| \le Y_j$, where $A_0, Y_1, \ldots, Y_n$ are positive constants.

2.2. LWL NN Approximation

To facilitate control design, the uncertain nonlinear function $f(\mathbf{x})$ is estimated by the following LWL NN:

$$\hat{f}(\mathbf{x}) = \frac{\sum_{k=1}^{N} w_k(\mathbf{x})\hat{f}_k(\mathbf{x})}{\sum_{k=1}^{N} w_k(\mathbf{x})}, \tag{4}$$

in which $w_k(\mathbf{x})$, $k = 1, \ldots, N$ are weighting functions, and $\hat{f}_k(\mathbf{x})$, $k = 1, \ldots, N$ are given by

$$\hat{f}_k(\mathbf{x}) = \hat{\theta}_k^T \phi_k(\mathbf{x}), \quad \phi_k(\mathbf{x}) = [1, (\mathbf{x} - c_k)^T]^T \tag{5}$$

with $\hat{\theta}_k$ and c_k the weight and center of the kth local estimator, respectively.

Assume $S_k = \{\mathbf{x} : w_k \neq 0\}$, $k = 1, 2, \ldots, N$ are a series of compact sets, which satisfy $D \subseteq \cup_{k=1}^{N} S_k$. Then, for any $\mathbf{x} \in D$, there exists at least one k such that $w_k \neq 0$.

Define the weighted functions $w_k(\mathbf{x})$ as

$$w_k(\mathbf{x}) = \begin{cases} \left(1 - (||\mathbf{x} - c_k||/\mu_k)^2\right)^2, & \text{if } ||\mathbf{x} - c_k|| \le \mu_k \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where μ_k is the radius of the kth local estimator. Let $\bar{w}_k(\mathbf{x}) = w_k(\mathbf{x})/\sum_k w_k(\mathbf{x})$ which satisfies $\sum_{k=1}^{N} \bar{w}_k = 1$. For ease of notation, the symbols ϕ_k, $\bar{w}_k$, f and $\hat{f}$ are used to represent the functions $\phi_k(\mathbf{x})$, $\bar{w}_k(\mathbf{x})$, $f(\mathbf{x})$ and $\hat{f}(\mathbf{x})$, respectively. Then, the locally weighted approximation (4) can be expressed as

$$\hat{f}(\mathbf{x}) = \sum_{k=1}^{N} \bar{w}_k \hat{f}_k(\mathbf{x}). \tag{7}$$

Define the optimal weight θ_k for $\mathbf{x} \in S_k$ as

$$\theta_k = \underset{\theta_k}{argmin} \left(\int_{\mathbf{x} \in D} w_k(\mathbf{x}) ||f(\mathbf{x}) - \hat{f}_k(\mathbf{x})||^2 dX \right) \tag{8}$$

and the local estimation error ϵ_k as

$$\epsilon_k = \begin{cases} f(\mathbf{x}) - \hat{f}_k(\mathbf{x}), & \text{on } \bar{S}_k \\ 0, & \text{on } D - \bar{S}_k \end{cases}$$

where $\bar{S}_k$ is the minimum compact set that containing S_k as a subset. Then, $f(\mathbf{x})$ and its NN estimator can be represented as

$$f = \sum_{k=1}^{N} \bar{w}_k \theta_k^T \phi_k + \sum_{k=1}^{N} \bar{w}_k \epsilon_k, \tag{9}$$

$$\hat{f} = \sum_{k=1}^{N} \bar{w}_k \hat{\theta}_k^T \phi_k \tag{10}$$

Then, the estimation error $\tilde{f} \triangleq f - \hat{f}$ can be expressed as

$$\tilde{f} = \sum_{k=1}^{N} \bar{w}_k \tilde{\theta}_k^T \phi_k + \sum_{k=1}^{N} \bar{w}_k \epsilon_k \tag{11}$$

with $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$. Assume $|\epsilon_k| \le \epsilon$ and $||\theta_k|| \le \theta_{max}$ with ϵ and θ_{max} being positive. Thus, one has $|\sum_{k=1}^{N} \bar{w}_k \epsilon_k| \le \max(|\epsilon_k|) \sum_{k=1}^{N_i} \bar{w}_k \le \epsilon$.

3. Control design and stability analysis

In this section, the state constraints and the asymmetric control saturation are tackled by considering control as an extended state and introducing a BLF in each step of the backstepping procedure. The system uncertainty $f(\mathbf{x})$ is approximated by a LWL approximator with weights updated by a composite error.

3.1. Locally weighted learning control

Step 1: Define $z_1 = x_1 - y_d$ as the trajectory tracking error, whose dynamics can be written as

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d. \tag{12}$$

Consider the BLF

$$V_1 = \frac{1}{2} \ln \frac{k_{b,1}^2}{k_{b,1}^2 - z_1^2} \tag{13}$$