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Geometrically nonlinear dynamic formulation for three-dimensional co-rotational solid elements

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Abstract

This study presents a three-dimensional nonlinear dynamic formulation based on a co-rotational (CR) approach for solid elements. The CR formulation is relatively efficient, but it is based on the assumption of small strains during large displacement. The novel idea of the present formulation involves the use of the CR formulation through a three-dimensional solid element for inertial quantities in addition to an internal force vector and a stiffness matrix. The present dynamic formulation is derived from Lagrange's equation of motion. In this procedure, the CR formulation, (i.e., element-independent CR) is one of the most attractive features that is strongly manifested in an efficient manner. Consequently, this obtains the governing equation of motion including motion-driven inertial components (physical quantities induced by the prescribed motion). Four examples are presented to demonstrate the accuracy of the present dynamic formulation. Finally, the results are compared with those obtained by ABAQUS, and the findings reveal that the proposed dynamic formulation is in good agreement with existing predictions.

Keywords: Nonlinear dynamic formulation; Co-rotational formulation; Three-dimensional solid element

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1. Introduction

A variety of structural behaviors ranging from small strains to large displacements are observed in engineering structural components. The accurate design and prediction of these behaviors has led to the development of relevant analytical models based on the finite element method. Extant studies developed a variety of finite element entities for

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three-dimensional structures. The various series of finite elements include solid-type elements that do not contain rotational degrees of freedom. Therefore, they could possess advantages related to simplicity and efficiency in formulation and modeling when compared to degenerated shell or beam elements. Additionally, three-dimensional solid elements, i.e., hexahedrons and tetrahedrons, are indispensable to the discretization of a structure with a complex configuration.

The volumetric feature in the elemental definition induces significant numerical problems such as locking problems. Specifically, findings indicate that solid type elements in a bending prominent problem possess severe locking problems [1]. Previous studies suggested various improvement efforts to solve this problem and included approaches that involved the use of an element with enhanced assumed strain (EAS) or assumed natural strain (ANS). The EAS elements were developed in several previous studies [1–5], and relevant solid elements related to the ANS approach were developed for an eight-node hexahedral element [6–12]. With respect to the geometrically nonlinear analysis, conventional Lagrangian formulation, i.e., total and updated Lagrangian description, were employed in a number of previous studies [13–20]. However, a conventional Lagrangian formulation for nonlinear analysis requires a significant modification in existing elemental hypothesis. Thus, the modifying procedure potentially becomes complicated with respect to the stress or strain definition [17].

A co-rotational (CR) formulation constitutes a relevant alternative to facilitate geometric nonlinearity. There is a growing amount of research interest in the CR formulation due to its efficiency for the problems under small strains and large rotations. In the earlier developments, the CR formulation was described by the terms, i.e., relevant Refs. [21,22], due to the existence of the CR frame located at each element which translates and rotates while covering the element. A feature of the element-independent nature of the CR formulation (EICR) was suggested by Rankin et al. [23]. An explicit review of the unified formulation was conducted by Felippa et al. [24]. The CR formulation was widely used in classical shell elements and others based on the continuum mechanics [25–30], and it was further extended to large strain problems [31]. Battini et al. proposed a relevant treatment of a rigid body component in a two-dimensional planar element [32]. This was extended to planar elements including in-plane rotational degrees of freedom [33]. Additionally, Eriksson et al. [34,35] proposed a CR formulation for a membrane in a three-dimensional space by extending Battini's approach [32].

Furthermore, extant studies proposed a CR formulation for three-dimensional volumetric solid elements [36–38]. Mostafa et al. developed a solid-shell element based on ANDES (assumed natural deviatoric strain), ANS, and EAS formulations. This type of linearized element was applied for geometrically nonlinear structural analysis by employing a CR formulation [38,39]. Nodargi et al. [40] developed a mixed four-node tetrahedral solid element. Moreover, a relevant extension for nonlinear analysis involved a CR formulation including the reference frame without any changes in the element node ordering scheme. More recently, a study proposed a consistent CR formulation that exploited the EICR feature, and a relevant verification was performed for tetrahedrons and hexahedrons [41].

Conversely, studies suggest that the derivation of the CR formulation for dynamic analysis is cumbersome owing to its complex nature [42]. Thus, several researchers employed conventional approaches such as a constant mass matrix [42–45]. Recently, Le et al. proposed a nonlinear dynamic formulation based on a CR formulation for two-dimensional and three-dimensional beam elements [46–48]. Especially, Le's formulation for two-dimensional beam analysis [46] was extended by Fharoughi et al. for a three-dimensional membrane [49]. However, Fharoughi's dynamic formulation was unable to maintain the EICR feature, which constitutes a fascinating point in the CR context.

In the present study, the dynamic formulation for three-dimensional solid elements is proposed by extending Le's formulation [46]. In the procedure, a local formulation is devised by adopting a Lagrangian description. Additionally, an element-independent CR is consistently maintained in an efficient manner for performing a three-dimensional dynamic analysis of solid elements. The relevant formulation includes nonlinear inertial terms, i.e., gyroscopic and centrifugal components. This results in obtaining the governing equation of motion including the dynamic quantities (nonlinear inertial terms and motion-driven terms). Four examples are evaluated to verify accuracy of the present analysis. This is followed by estimating both accuracy as well as efficiency of the present analysis.

2. Elemental kinematics

Elemental coordinates in the CR formulation are defined to track an elemental motion. The relevant coordinates in the present formulation include a global fixed frame, an undeformed frame, and a corotated frame. These coordinates are defined with respect to the body-fixed floating frame to consider the prescribed motion. The resulting coordinates and elemental kinematics of the solid element are shown in Fig. 1. An eight-node solid is depicted for convenience.