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Variational approach to interface element modeling of brittle fracture propagation

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Abstract

The paper proposes a variational approach to model brittle fracture propagation based on zero-thickness finite elements. Similar to the phase-field model for fracture, the problem of a fractured structure is variationally formulated by considering a minimization problem involving bulk and fracture surface energies. With the help of a damage variable used as an additional degree of freedom, the fracture propagates according to the values of the minimizers of the total potential energy. This damage variable is restricted to dimensionally reduced interface elements inserted between element boundaries. Crack opening is predicted when the elastic energy within the interface surface exceeds the critical energy release rate. The solution of the discretized system of equations is performed in a staggered scheme, solving first for the displacement field and then searching for the solution for the updated nodal damage variables. Selected numerical examples, including re-analyses of laboratory tests characterized by rather complex crack paths, are presented to demonstrate the performance of the proposed variational interface model.

Keywords: Variational method; Interface element; Fracture mechanics; Brittle fracture; Phase-field model; Finite element method

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1. Introduction

Failure of structures made of brittle and quasi-brittle materials, such as glass, ceramics, concrete or geological materials, is generally induced by the initiation and propagation of cracks and often characterized by complex fracture patterns associated with crack branching and coalescence of different crack segments. Numerical simulations of fracture processes in such materials require robust computational models able to represent the discontinuous character of the cracking process and to predict crack initiation and propagation as well as possible coalescence and branching phenomena.

Fracture in brittle solids is generally formulated in the framework of the theory of Linear Fracture Mechanics (LEFM), which is based on the idealization of cracks as fully separated fracture surfaces associated with a singular

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stress distribution at the crack tip [1]. Early finite element models were representing brittle fracture by a separation of elements along common edges in conjunction with singular crack tip elements [2–4], often combined with adaptive meshing methods [5]. Later, the Extended Finite Element Method (XFEM) allowed to include discontinuous displacement fields directly into finite elements using the Partition of Unity property of finite elements along with suitable enrichment functions at the crack tip and across open cracks, respectively (see, e.g. [6,7]). In these models, the direction of the next crack segment is evaluated from criteria based on the evaluation of the stress intensity factors. A variational formulation of crack propagation using XFEM was proposed in [8] by including the angle of crack propagation and the crack length as unknown variables into the variational statement, computing the crack path by solving an energy minimization problem. As an alternative to the explicit modeling of fracture surfaces, Pandolfi and Ortiz [9] proposed a fracture model, denoted as Eigenerosion approach, which is based on the deletion of volume elements such that the fracture energy is preserved. In this model, the fracture criterion is checked non-locally within a given radius according to the maximum dissipation energy, which converges to the Griffith criterion as the mesh size approaches zero.

In contrast, in models designed for cohesive-frictional materials, a cohesive zone, characterized by evolving microcracks in the crack tip region, is assumed, in which residual tractions gradual decrease. Cohesive zone models were introduced by Barenblatt [10] and found wide spread applications in the simulation of quasi-brittle materials, which are characterized by a process zone instead of a sharp crack tip (see, e.g. [11] for an extensive overview). In cohesive models, the dissipated energy $\mathcal{G}$ increases with the increase of the crack opening, which is governed by a cohesive traction-separation law, providing a relation between normal and tangential traction forces and crack displacements, respectively. Besides continuum based damage models (see, e.g., [12,13] for an overview), discrete crack models equipped with cohesive traction-separation laws have been proposed using different discretization approaches. One popular method are interface models, in which zero-thickness elements are inserted along the edges of solid finite elements [14]. This class of models has been successfully used in meso-scale analyses of concrete materials [15] as well as to applications on the structural scale [16]. An efficient format in regards to implementation was proposed by Manzoli et al. [17], where the zero-thickness interface elements are substituted by solid interface elements with high aspect ratio. The solid interface element approach was recently applied to fiber reinforced concrete structures by Zhan and Meschke [18]. Zero-thickness interface elements are also extensively used in geomechanical applications in order to model joints in rock masses. Here, the representation of the sliding characteristics is accomplished by joint friction models [14,19,20]. The increased computational demand resulting from inserting interface elements between solid elements can be controlled by pre-defining the interface elements only in vulnerable regions [21] or applying adaptive algorithms for mesh processing during computation [22–24].

Nevertheless, cohesive and frictional interface elements possess a number of numerical difficulties. Schellekens and De Borst [25] discussed the oscillatory behavior of cohesive elements using the standard Gauss-point integration rule which can be reduced by utilizing alternative integration schemes. Later, De Borst [26] published work on the mesh biased influence on the fracture propagation using the interface elements. He proposed a method to overcome this drawback by the enrichment of finite element shape functions by means of the partition-of-unity method. Problems associated with the numerical stiffness of the interface elements and softening behavior of the cohesive law are discussed in [27–29].

The advantage of variational formulations in obtaining solutions for crack propagation problems on a structural level, which are not relying on local criteria, has motivated the development of variational models for brittle fracture [30,31], later re-formulated as phase-field models by Miehe et al. [32], Kuhn and Müller [33], Borden et al. [34], Ambati et al. [35], among many others. In this class of models, cracks are represented as continuous damage zones in terms of a field variable c instead of discrete displacement discontinuities. The width of the damage zone is defined by a numerical fracture length parameter l_c. This length parameter, however, cannot be interpreted and calibrated independently, e.g. as the size of the fracture zone is related to the heterogeneity of the material (as was determined e.g. by Le Bellégo et al. [36]), since it also affects the simulated strength of the material. Therefore, l_c must be calibrated according to the tensile strength of the material [37]. It is noted, that phase-field models are closely related to gradient damage models [38] and, since l_c has to be larger than the mesh size h, require a fine resolution of the damage zone across its width, which may lead to an enormous discretization effort.

In this paper, we propose a variational formulation for fracture in non-cohesive, brittle materials, which is inspired by the phase-field model, but which is incorporated into a zero-thickness interface modeling framework instead of a continuum damage framework. This approach reduces the discretization effort by orders of magnitude as compared to