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Research paper

Fast hyperbolic Radon transform represented as convolutions in log-polar coordinates

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ABSTRACT

The hyperbolic Radon transform is a commonly used tool in seismic processing, for instance in seismic velocity analysis, data interpolation and for multiple removal. A direct implementation by summation of traces with different moveouts is computationally expensive for large data sets. In this paper we present a new method for fast computation of the hyperbolic Radon transforms. It is based on using a log-polar sampling with which the main computational parts reduce to computing convolutions. This allows for fast implementations by means of FFT. In addition to the FFT operations, interpolation procedures are required for switching between coordinates in the time-offset; Radon; and log-polar domains. Graphical Processor Units (GPUs) are suitable to use as a computational platform for this purpose, due to the hardware supported interpolation routines as well as optimized routines for FFT. Performance tests show large speed-ups of the proposed algorithm. Hence, it is suitable to use in iterative methods, and we provide examples for data interpolation and multiple removal using this approach.

1. Introduction

In the processing of Common-Midpoint gathers (CMPs), the hyperbolic Radon transform has proven to be a valuable tool for instance in velocity analysis (Clayton and McMechan, 1981; Greenhalgh et al., 1990); aliasing and noise removal (Turner, 1990); trace interpolation (Averbuch et al., 2001; Yu et al., 2007); and attenuation of multiple reflections (Hampson, 1986). The hyperbolic Radon transform is defined as

$$\mathcal{R}_{\mathrm{h}} f(\tau, q) = \int_{-\infty}^{\infty} f(\sqrt{\tau^2 + q^2 x^2}, x) dx, \tag{1}$$

where the function $f(t, x)$ usually corresponds to a CMP gather. Here, the parameter q characterizes an effective velocity value; and τ represents the intercept time at zero offset.

Several versions of Radon transforms are used in seismic processing, e.g., straight-line, parabolic, and hyperbolic Radon transforms. In many applications there is a need for a sparse representation of seismic data using hyperbolic wave events. One way to get sparse representations is by using iterative thresholding algorithms with sparsity constraints (Daubechies et al., 2004; Sacchi and Ulrych, 1995). Popular applications using such representations are seismic data interpolation and wavefield separation (Jiang et al., 2016; Trad, 2003). Since iterative schemes for computing such representations require the application of the forward and adjoint operators several times, it becomes important to use fast algorithms to limit the total computational cost.

Note that the direct summation over hyperbolas in (1) has a computational complexity of $O(N^3)$, given that the numbers of samples for the variables t, x, τ, q are $O(N)$. There are many effective ($O(N^2 \log N)$) methods for rapid evaluation of the traditional Radon transforms, or the parabolic Radon transform, see Beylkin (1984); Fessler and Sutton (2003); Schonewille and Duijndam (2001). The hyperbolic Radon transform is, however, more challenging. Nonetheless, a fast ($O(N^2 \log N)$) method for hyperbolic Radon transforms was recently presented in Hu et al. (2013). The method is based on using the fast butterfly algorithms described in O'Neil (2007) and Candès et al. (2009), and versions addressing computational efficiency are presented in Poulson et al. (2014), Li et al. (2015b, 2015a) and Li and Yang (2016).

A fast method for the standard Radon transform was proposed in Andersson (2005) by expressing the Radon transform and its adjoint in terms of convolutions in log-polar coordinates. To use this approach, it is necessary to resample data in log-polar coordinates, and this requires some interpolation method. An important property of such a scheme is that since the interpolation procedures are performed in the

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Radon and image domains, the errors will be kept local. On the contrary, the interpolation errors in the frequency domain will create global non-systematic errors when switching back to the time-space domain. Since these non-localized errors contribute to the general structure of the function, they have to be mainly controlled.

Computationally efficient algorithms for GPUs were presented for the log-polar-based approach in Andersson et al. (2016). In this paper we propose to use the same approach and construct algorithms with complexity $O(N^2 \log N)$ for evaluation of the hyperbolic Radon transform. We present computational performance tests confirming the expected accuracy and the computational complexity, as well as predicted computational speed-ups for parallel implementations. Finally, we present several synthetic and real data tests using the hyperbolic Radon transform for data interpolation and multiple attenuation.

2. Method

To begin with, we note that functions $f(t, x)$ describing CMP gathers are symmetric with respect to x=0. Hence, by introducing

$$\widetilde{f}(s, y) = \frac{f(\sqrt{s}, \sqrt{y})}{2\sqrt{y}}, \tag{2}$$

it follows that

$$\mathcal{R}_{\rm h} f(\tau, q) = 2\int_0^\infty f(\sqrt{\tau^2 + q^2x^2}, x)dx = 2\int_0^\infty \widetilde{f}(\tau^2 + q^2y, y)dy. \tag{3}$$

The resulting expression in (3) has a form of the Radon transform over straight lines, and a fast algorithm for the evaluation of this was presented in Andersson et al. (2016), referred to as the log-polar Radon transform which is based on rewriting the key operations as convolutions in a log-polar coordinate system. In Section 2.1 we briefly recall the construction of the log-polar Radon transform and discuss how to adjust this method for optimal performance when processing seismic data, and in Section 2.2 we introduce coordinate transforms as well as sampling/interpolation requirements for accurate evaluation of $\mathcal{R}_{\rm h} f(\tau, q)$.

The fact that the hyperbolic Radon transform can be computed by a combination of a change of variables along with an application of the regular Radon transform implies that it can be evaluated using many different methods for computing the regular Radon transform, including methods with an hierarchical decomposition of the Radon transform (Basu and Bresler, 2000; George and Bresler, 2007), methods based on Fourier slice theorem (Beylkin, 1995; Fessler and Sutton, 2003), or chirp-Z transforms (Averbuch et al., 2006). In Andersson et al. (2016) a thorough comparison between fast implementations of Radon transform is presented, and where the log-polar implementation has an advantage with respect to computational performance in comparison to other approaches.

Another important aspect is the computational accuracy. Depending on how large approximation errors that are acceptable, different computational speedups can be achieved. For some applications rather low level of accuracy (i.e., errors of say 10%) could be acceptable. This could for instance be the case of event detection (but not removal). In this paper, we aim at keeping an accuracy level of a couple of digits, since this seems to be enough for most seismic applications. The accuracy level will be dependent on the interpolation error, and we show that this error can be reduced if higher order interpolation kernels are used.

Different algorithms also have different behavior in terms of how interpolation errors propagate. In this regards, the proposed method has an advantage, since the interpolation takes place in the measurement domain rather than in the frequency domain, which will keep errors more localized.

2.1. Log-polar Radon transform

The standard Radon transform (cf. (3)) can be written in terms of a double integral

$$\mathcal{R}\widetilde{f}(\tau^2, q^2) = \int\int \widetilde{f}(s, y)\delta(s - \tau^2 - q^2y)dyds, \tag{4}$$

where δ denotes the Dirac distribution. In Andersson et al. (2016) one works with the log-polar coordinates

$$\begin{cases} s = e^{\rho'}\cos(\theta'), \\ y = e^{\rho'}\sin(\theta'), \end{cases} \quad \begin{cases} \tau^2 = \dfrac{e^{\rho}}{\cos(\theta)}, \\ q^2 = -\tan\theta. \end{cases} \tag{5}$$

By introducing $\zeta(\theta, \rho) = \delta(\cos(\theta) - e^{\rho})$, it turns out that the Radon transform can be efficiently evaluated using the *log-polar Radon transform*

$$\mathcal{R}_{\rm lp}\widetilde{f}(\theta, \rho) = \cos(\theta)\int\int \widetilde{f}(\theta', \rho')e^{\rho'}\zeta(\theta - \theta', \rho - \rho')d\rho' d\theta'$$

where, by abuse of notation, we use the same notation $\widetilde{f}$ for both coordinate representations.

However, the above representation is not suitable for treating functions $\widetilde{f}$ with support near the origin, since the origin is represented by $\rho = -\infty$. A way around this obstacle is to make a translation so that the support of $\widetilde{f}$ is moved away from the origin, and then work with functions supported within a subset of a unit circle-sector of opening angle β as in Fig. 2, right. In this figure the procedures of scaling and rotations are also applied in order to make a proper fit and connection to the interval for variable q from (4). The scale parameter, the rotation angle, as well the opening angle β will be defined in what follows. With this setup, the function $\widetilde{f}$ in log-polar coordinates and its log-polar Radon transform $\mathcal{R}_{\rm lp}\widetilde{f}(\theta, \rho)$ both have compact support. In this work we consider a simplified version with one circle-sector, because the hyperbolic Radon transform is typically computed for some small interval in the slowness variable q, which is directly related to the angle β. In contrast, three circle-sectors were used in Andersson et al. (2016), see Fig. 3. This is because that paper was aimed at treating data arises from line integrals for all directions, and in this case it is necessary to split the directional interval into at least three parts. The log-polar Radon transform is computed for each of these parts. Computations for all three intervals were done by using a formula for symmetric intervals, cf. (Andersson et al., 2016, Formulas 2.5, 2.6), where preliminary rotation procedures were applied to process non-symmetric intervals, see transformations T_m and S_m from Andersson et al. (2016), Formulas 2.9, 2.12. The final formula for computing the Radon transform by using preliminary rotations and the log-polar Radon transform for the symmetric interval reads as (Andersson et al., 2016, Formula 2.13).

To make this work consistent with our previous paper (Andersson et al., 2016), we deal with a symmetric interval for variable θ, i.e. $\theta \in [-\beta/2, \beta/2]$. Note, that other non-symmetric intervals of size β can be considered as alternatives, but after rotation procedures (similar to transformations T_m and S_m mentioned above) they would give the same end result as using a symmetric interval to begin with. We will refer to the implementation of $\mathcal{R}_{\rm lp}$ for values $\theta \in [-\beta/2, \beta/2]$ as the algorithm of *partial* $\mathcal{R}_{\rm lp}$.

With this in mind, we now briefly explain how to make slight modifications to the above scheme, better suited for the processing of CMP gathers. A simplified synthetic example of a typical CMP gather is shown in Fig. 1. Note that the function continues outside the maximum limits given by x and t, leading to a truncation of (3), (which can be seen e.g. as the circular artifacts in Fig. 1). Also note that there is no data in the region above a line t=kx, i.e. high offset x and small time intercept t, so to decrease the amount of computations we may ignore this piece. In the coordinates (s,y) this triangle is again a triangle, but with equation $s = k^2y$. We set $\gamma = \arctan k^2$. Thus, we are in practice only