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Development of a direct time integration method based on Bezier curve and 5th-order Bernstein basis function

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ABSTRACT

In this study, a robust unconditionally stable method for linear analysis of structures based on Bezier curves and Bernstein polynomials is proposed. The Bezier curve is used as interpolation function and Bernstein basis functions are applied for interpolation. The spectral radius, period elongation and amplitude decay are investigated for stability analysis, numerical dispersion and dissipation of proposed method, and results are compared with other methods that are the best in these properties. It is also shown that the behavior of the proposed method in analysis of finite element system is effective and reliable. To show the robustness and features of proposed method, a challenging problem with a very stiff and flexible response, a Howe truss under impact load, a frame under harmonic loading and a rectangular domain in plane strain condition are considered, and derived results are compared with references solutions and other results reported in the literature.

1. Introduction

According to the importance of numerical methods in solving equations of structural dynamics, much research has been done and different time integration methods for linear and nonlinear analysis of structures such as Newmark-β [1], Wilson-θ [2], Houbolt [3], HHT-α [4], Bathe [5] and Park [6] are reported in the literature. The stability and accuracy of time integration methods have been discussed and studied in Refs. [7–9]. Integration algorithms can be categorized into two main classes: explicit methods [10–14] and implicit ones [1–5]. Explicit methods need less computational effort than the implicit ones. This matter has been fully investigated in Refs. [15,16].

In addition to the afore-mentioned classes, another classification based on unconditionally stable algorithms or conditionally stable ones can be pointed out. In conditionally stable algorithms [11–13], it is necessary to apply the time step whose size is inversely proportional to the highest frequency of the discrete systems. In other words, it is needed to use a time step which is less than the smallest period of the interested structure. In complex structural models, this restriction is a difficulty, especially when the response of lower mode is interested because much smaller time steps than the ones needed for accuracy are required. In unconditionally stable algorithms [17–22], choosing the size of time step is independent of stability that leads to reducing the computational effort in the analysis. Another feature of these algorithms can be numerical dissipation for suppress spurious participation of the higher modes, while an unconditionally stable algorithm may not have numerical dissipation, for example the Newmark method (trap. rule) [1]. Therefore, unconditionally stable algorithms are preferred more than conditionally stable ones (for more details, see Refs. [4,8]).

In addition to unconditionally stable feature, dependable and effective behavior in solution of the equation of dynamic equilibrium of finite element system is one of the valuable properties of numerical methods in linear analysis of structural dynamic problems. Although the finite element equations illustrate special properties, the time integration methods should have dependable behavior in analysis of them, especially in exigent large deformation solutions [23–26]. One of the features that make appropriate behavior in analysis of finite element systems is numerical dissipation. In fact, an unconditionally stable algorithm must somehow acts in a way that in addition to dissipation spurious participation of the higher modes, it should not affect the lower modes too strong that incurs a substantial loss of accuracy. Bathe and Noh [5] offered an implicit time integration scheme with unconditional stability that uses two sub-steps inside each time step that shows a good performance in finite element systems.

In this study, the time integration approach is reconsidered by proposing Bernstein polynomials and Bezier curve to approximate displacement, velocity and acceleration fields in dynamic of structures. Our objective in this study is to present a robust, high-

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efficient, and unconditionally stable method in linear analysis without the use of adjustable parameters and investigate its properties. The present method is based on Bezier curve as interpolation function and Bernstein basis functions for interpolation. The stability and accuracy of the present method are investigated with the use of amplification matrix and its eigenvalues.

2. Bezier curve definition

A Bezier curve is a parametric curve that utilizes the Bernstein polynomials as a basis function. A Bezier curve of degree n is expressed by

$$P(t) = \sum_{i=0}^{n} J_{n,i}(t) b_i \tag{1}$$

where b_i are the Bezier points or control points and $J_{n,i}(t)$ is the ith nth-order Bernstein basis function, that n is the degree of the Bernstein basis function [27]. $J_{n,i}(t)$ in interval $[t_j, t_{j+1}]$ is represented by

$$J_{n,i}(t) = \binom{n}{i} (t_{j+1} - t_j)^{-n} (t_{j+1} - t)^{n-i} (t - t_j)^i, \quad t \in [t_j, t_{j+1}]$$
$$\binom{n}{i} = \frac{n!}{i!(n-i)!}, \quad 0! \equiv 1, \quad (0)^0 \equiv 1, \quad i = 0, \dots, n \tag{2}$$

In this study, Bernstein basis functions are used for interpolation and Bezier curves are employed as interpolation function.

3. Preparing Bernstein basis functions for interpolation

The first and second order derivatives of Bernstein basis functions in interval $[t_j, t_{j+1}]$ are required for the interpolation, which are obtained as follows:

$$J^{(1)}_{n,i}(t) = \binom{n}{i} (t_{j+1} - t_j)^{-n} \big(i (t_{j+1} - t)^{n-i} (t - t_j)^{i-1} - (n-i)(t_{j+1} - t)^{n-i-1} (t - t_j)^i\big), \quad t \in [t_j, t_{j+1}] \tag{3}$$

$$J^{(2)}_{n,i}(t) = \binom{n}{i} (t_{j+1} - t_j)^{-n} \big((n-i-1)(n-i)(t_{j+1} - t)^{n-i-2} (t - t_j)^i - 2i(n-i)(t_{j+1} - t)^{n-i-1} (t - t_j)^{i-1} + i(i-1)(t_{j+1} - t)^{n-i} (t - t_j)^{i-2}\big), \quad t \in [t_j, t_{j+1}] \tag{4}$$

To prepare the Bernstein basis functions for the interpolation, the intervals of Eqs. (2)–(4) had to be changed from $[t_j, t_{j+1}]$ to $[0, 1]$.

To change the parameter from $t \in [t_j, t_{j+1}]$ to $\tau \in [0, 1]$, $\tau = \frac{(t - t_j)}{(t_{j+1} - t_j)}$ can be used, which gives $t = \tau(t_{j+1} - t_j) + t_j$, therefore:

$$J(t), t \in [t_j, t_{j+1}] \equiv J(\tau(t_{j+1} - t_j) + t_j), \quad \tau \in [0, 1] \tag{5}$$

If the distance between t_j until t_{j+1} is considered a time step, then:

$$t = \tau(\Delta t) + t_j \tag{6}$$

By replacing Eq. (6) into Eqs. (2)–(4), Bernstein basis functions are obtained for interpolation as below:

$$J_{n,i}(\tau) = \binom{n}{i} (1 - \tau)^{n-i} (\tau)^i, \quad \tau \in [0, 1] \tag{7}$$

$$J^{(1)}_{n,i}(\tau) = \frac{1}{\Delta t} \binom{n}{i} \big(i(1 - \tau)^{n-i} (\tau)^{i-1} - (n-i)(1 - \tau)^{n-i-1} (\tau)^i\big), \quad \tau \in [0, 1] \tag{8}$$

$$J^{(2)}_{n,i}(\tau) = \frac{1}{\Delta t^2} \binom{n}{i} \big((n-i-1)(n-i)(1 - \tau)^{n-i-2} (\tau)^i - 2i(n-i)(1 - \tau)^{n-i-1} (\tau)^{i-1} + i(i-1)(1 - \tau)^{n-i} (\tau)^{i-2}\big), \quad \tau \in [0, 1] \tag{9}$$

4. Interpolation method based on the Bezier curve of degree 5

A set of basis functions of degree 5 as well as their first and second order derivatives that are used for interpolation, are shown in Figs. 1–3.

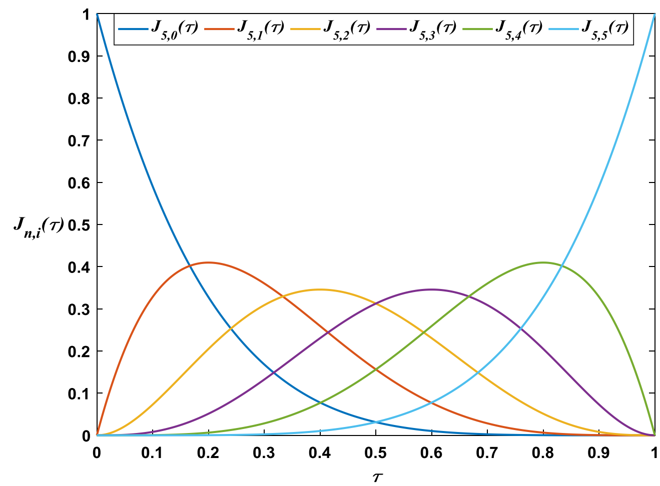

Fig. 1. The prepared Bernstein basis functions.

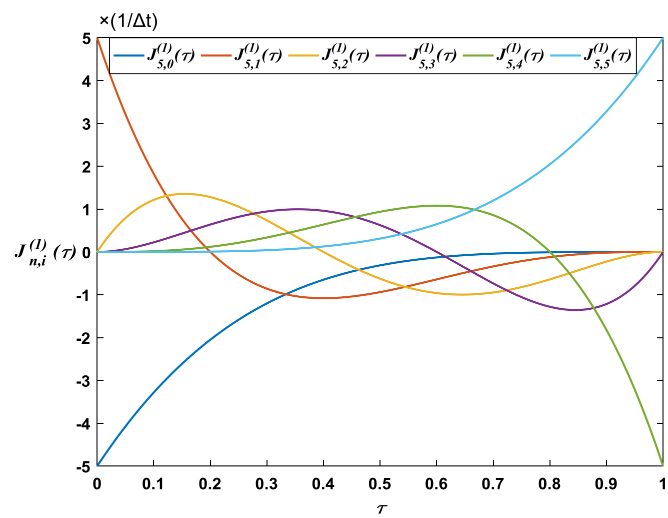

Fig. 2. The prepared first-order derivative Bernstein basis functions.

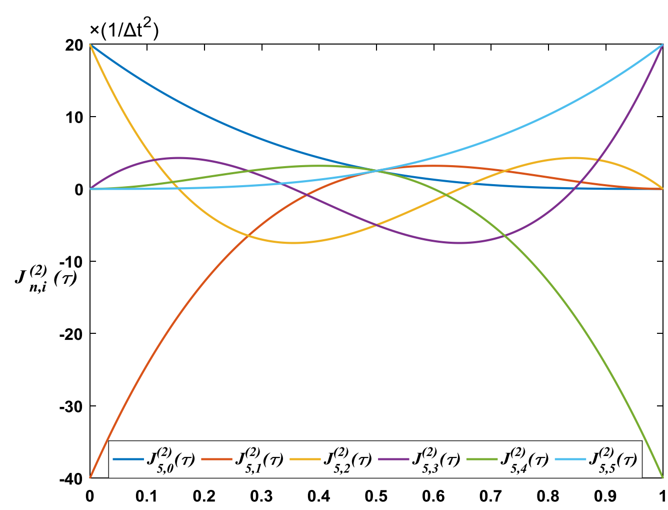

Fig. 3. The prepared second-order derivative Bernstein basis functions.