Computers and Structures 194 (2018) 48–59

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Roles of load temporal correlation and deterioration-load dependency in structural time-dependent reliability

Cao Wang, Hao Zhang *

School of Civil Engineering, The University of Sydney, Sydney, NSW 2006, Australia

A R T I C L E   I N F O

Article history:
Received 15 March 2017
Accepted 12 September 2017
Available online 22 September 2017

Keywords:
Time-dependent reliability
Temporal correlation
Stochastic load process
Shock deterioration
Deterioration-load dependency
Safety assessment

A B S T R A C T

Aging of structural performance and significant external loads may impair structural safety and serviceability and cause potential economic losses. In the presence of uncertainties associated with both resistance deterioration and external loads, structural safety shall be estimated quantitatively under a probability-based framework. A stochastic load process is often auto-correlated on the temporal scale, with correlations arising from both the occurrence times and intensities. Moreover, a deterioration process is physically dependent on the load magnitudes. This paper investigates the impacts of load temporal correlation and deterioration-load dependency on time-variant structural reliability. The load occurrence process is modeled as a Poisson point process with correlated separation time between two load events. The correlation between the intensities of load events is described by the multivariate Gaussian copula function. The resistance aging process is considered to be a combination of both gradual and shock deteriorations. Four candidate copula functions, namely Gaussian, Clayton, Gumbel and Frank, are considered to model the dependency of shock deterioration on load intensity. Two types of failure mechanisms are considered: the first is due to the load effect exceeding the resistance, and the second occurs when the cumulative damage within the considered service period reaches the permissible level. A simulation-based method is developed to estimate structural reliability considering the two failure modes. Illustrative examples are presented to demonstrate the applicability of the proposed method. Parametric studies are conducted to investigate the impacts of temporal correlation in loads and deterioration-load dependency on structural failure probability.

1. Introduction

The performance of civil structures such as strength, stiffness and stability may deteriorate due to severe operating or environmental conditions in service, resulting in a potential decrease of structural safety and serviceability below the baseline as assumed for new ones. In an attempt to achieve a better understanding of the service conditions of aging structures, it is of significant importance to assess the structural safety and remaining service-life under a probability-based framework, taking into account the uncertainties associated with both the resistance deterioration and load process [1–4]. Structural reliability is a widely-used indicator of structural ability to fulfill the safety and serviceability requirements during a specific time period of interest, and provides a rational criterion to help make decisions regarding the maintenance optimizations of structures [5–8].

Significant studies have been conducted in recent decades regarding the time-variant reliability and service-life assessment of aging structures [9–16]. Mori and Ellingwood [9] proposed a closed-form solution for structural time-dependent reliability analysis considering a stationary load process. Later studies used this method to assess the remaining service life of aging structures [6,7,11]. Li et al. [4] improved the work by Mori and Ellingwood [9] and developed a method for reliability analysis of aging structures, which enables the non-stationarity in loads [17–19] to be considered. However, many previous works used a fully-correlated deterioration model, which cannot not fully address the stochastic characteristics associated with the deterioration process. Some improved deterioration models have later been proposed [20–27], where the monotonicity (non-increasing) and auto-correlation of the deterioration process are taken into account. Yet limited attention has been paid to the modeling of physical dependence of deterioration process on load intensities. This is particularly relevant when only incomplete statistical information is available. Furthermore, existing works have, for the most part, considered the load process as independent. Practically, temporal

* Corresponding author.
E-mail addresses: cao.wang@sydney.edu.au (C. Wang), hao.zhang@sydney.edu.au (H. Zhang).

http://dx.doi.org/10.1016/j.compstruc.2017.09.001
0045-7949/

correlation often exists between the load intensities and/or in the load occurrence due to common causes. For instance, for the tropical cyclone winds, at a specific site of interest, inter-correlation between successive cyclone events may be posed by common underlying climatological causes [28–30]. The observation of multi-year and multidecadal oscillations in cyclones also suggests such a temporal correlation [31]. Ellingwood and Lee [28] quantitatively measured the autocorrelation in wind load process, where a time series model [32] was used. Li et al. [29] and Wang et al. [30] preliminarily investigated the impact of temporal correlation in cyclone process on cumulative community damage. However, to-date methods for structural reliability analysis have yet to incorporate the temporal correlation in external load process.

This paper assesses the time-dependent reliability of aging structures in the presence of temporal correlation in loads and deterioration-load dependency. The correlations arising from both the load occurrence times and load intensities are taken into account. The dependency of resistance deterioration on load intensity is described by a copula function. A simulation-based method is developed for structural time-dependent reliability analysis. The proposed method is demonstrated through a time-dependent reliability assessment problem. The impacts of load temporal correlation and deterioration-load dependency on structural safety are investigated parametrically.

2. Modeling the temporal correlation in loads

Both the occurrence and intensity of significant loads are unavoidably associated with uncertainties on the time scale. As a result, they should be modeled using a probabilistic method. The mathematical modeling of a correlated load process is discussed in this section.

2.1. Temporal correlation in load occurrence

In practice, a stochastic random process such as the Poisson process is used to account for the randomness in load occurrence times [4,9,33]. For a reference period of T years, the loads can be represented by a sequence of randomly occurring pulses with random intensities, $S_1, S_2, \ldots S_N$, at times $t_1, t_2, \ldots t_N$, respectively. The sequence of time interval between two subsequent events, $\boldsymbol{\Delta} = \{\Delta_1, \Delta_2, \ldots \Delta_N\}$, can be used to define a Poisson process, where $\Delta_i = t_i - t_{i-1}$ for $i = 1, 2, \ldots N$ and $t_0 = 0$. It is an independent process and the cumulative density function (CDF) of Δ_i, F_{Δ_i}, is given by

$$F_{\Delta_i}(t) = 1 - \exp\left[-\int_0^t \lambda(t_{i-1} + \tau)\mathrm{d}\tau\right], \quad t \geqslant 0. \tag{1}$$

where $\lambda(\tau)$ is the mean occurrence rate of the load at time τ (i.e., on average $\lambda(\tau)$ load event(s) occur during unit time corresponding to time τ). Eq. (1) simply becomes $F_{\Delta_i}(t) = 1 - \exp(-\lambda t)$ for the case of a stationary process.

Now we consider the temporal correlation in load occurrence. The time interval sequence $\boldsymbol{\Delta}$ is modeled as a correlated Markov chain, that is, Δ_{i+1} is directly correlated with Δ_i only (see, e.g., [34] for details on Markov chain process). Let ϱ_i denote the linear correlation coefficient between Δ_i and Δ_{i+1}, with which the correlation coefficient between Δ_i and Δ_j is given by

$$\rho_{ij} = \begin{cases} \prod_{k=j}^{i-1} \varrho_k, & i > j \\ 1, & i = j \\ \rho_{ji}, & i < j \end{cases} \tag{2}$$

Specifically, if $\varrho_i \equiv \varrho$ for $\forall i$, ρ_{ij} in Eq. (2) becomes ϱ^{i-j} if $i > j$. The number of loads within time interval $(0, T]$, $N(T)$, is given by

$$N(T) = \max\left\{k : \sum_{j=0}^{k} \Delta_j \leqslant T\right\} \tag{3}$$

where $\Delta_0 = 0$. The validity of this correlated load process is guaranteed by the convergence of $\lim_{T\to\infty} \frac{N(T)}{T}$, as discussed in Appendix A. The Nataf transformation method [35–37] can be used to generate a sample sequence of $\boldsymbol{\Delta}$, provided the marginal distribution of each Δ_i and the correlation matrix $\boldsymbol{\rho} = [\rho_{ij}]$ are known. The basic idea is to first transform $\boldsymbol{\Delta}$ into a correlated standard normal distributed vector $\mathbf{Y} = \{Y_1, Y_2, \ldots Y_N\}$ with a correlation matrix of $\boldsymbol{\rho}' = [\rho'_{ij}]$, and then transform $\mathbf{Y}$ into an independent standard normal distributed vector $\mathbf{Z} = \{Z_1, Z_2, \ldots Z_N\}$. It can be shown that

$$\mathbf{Y} = \mathbf{L} \cdot \mathbf{Z} \tag{4}$$

where $\mathbf{L} = [l_{ij}]$ is a lower triangle matrix satisfying $\mathbf{L} \cdot \mathbf{L}^{\mathrm{T}} = \boldsymbol{\rho}'$.

A key step in the Nataf transformation method is to find the correlation matrix $\boldsymbol{\rho}'$ provided $\boldsymbol{\rho}$. Discussions on the relationship between ρ_{ij} and ρ'_{ij} can be found in literature (see, e.g., [35,36]). For the case of a stationary process, each Δ_i is identically distributed and follows an exponential distribution. The mean value and variance of Δ_i are $1/\lambda$ and $1/\lambda^2$ respectively, yielding a constant COV (coefficient of variation) of 1. With this, the relationship between ρ_{ij} and ρ'_{ij} can be found numerically as

$$\frac{\rho'_{ij}}{\rho_{ij}} = -0.0553\rho_{ij}^3 + 0.152\rho_{ij}^2 - 0.3252\rho_{ij} + 1.2285. \tag{5}$$

Note that expanding Eq. (4) gives

$$\begin{aligned} \Delta_1 &= F_{\Delta_1}^{-1}[\Phi(l_{11} \cdot Z_1)] \\ \Delta_2 &= F_{\Delta_2}^{-1}[\Phi(l_{21} \cdot Z_1 + l_{22} \cdot Z_2)] \\ &\vdots \\ \Delta_N &= F_{\Delta_N}^{-1}[\Phi(l_{N1} \cdot Z_1 + l_{N2} \cdot Z_2 + \ldots + l_{NN} \cdot Z_N)] \end{aligned} \tag{6}$$

where

$$l_{ii} = \sqrt{\rho'_{ii} - \sum_{k=1}^{i-1} l_{ik}^2} \tag{7}$$

and

$$l_{ij} = \frac{1}{l_{ii}}\left(\rho'_{ij} - \sum_{k=1}^{i-1} l_{ik} l_{jk}\right), \quad j \geqslant i + 1. \tag{8}$$

With this, the procedure of sampling a stationary and correlated sequence of $\boldsymbol{\Delta}$, $\{\delta_1, \delta_2, \ldots \delta_N\}$, by means of the Nataf transformation method is summarized as follows:

(1) Determine $\boldsymbol{\rho}'$ with Eq. (5), and solve $\mathbf{L}$ with Eqs. (7) and (8).
(2) Generate N independent standard normal distributed samples $z_1, z_2, \ldots z_N$.
(3) Set $\delta_i = F_{\Delta_i}^{-1}\left[\Phi\left(\sum_{j=1}^{i} l_{ij} z_j\right)\right]$ for $i = 1, 2, \ldots N$.

For the case of a non-stationary correlated process, however, the COV of Δ_i varies with i since it depends on t_{i-1}. This fact, unfortunately, indicates that one cannot construct the correlation matrix $\boldsymbol{\rho}'$ prior to generating a sample of $\boldsymbol{\Delta}$. In such a case, an iteration-based method is proposed to sample $\boldsymbol{\Delta}$.

Note that Eqs. (7) and (8) demonstrate that the elements in $\mathbf{L}$, l_{ij} $(i \geqslant j)$, are uniquely determined once the principal sub-matrix of $\boldsymbol{\rho}'$, $\boldsymbol{\rho}'[i, i]$, is known (see [38] for the definition of principal sub-matrix). With this, one can generate a sample sequence of $\boldsymbol{\Delta}$ as follows: