Computers and Structures 194 (2018) 60–73

Contents lists available at ScienceDirect

# Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

# A modal projection-based reduction method for transient dynamic responses of viscoelastic systems with multiple damping models

Zhe Ding [a,c,*], Li Li [b], Jianyi Kong [a,c], Li Qin [a,c]

[a] Key Laboratory of Metallurgical Equipment and Control Technology, Ministry of Education, School of Machinery and Automation, Wuhan University of Science and Technology, Wuhan 430081, China
[b] State Key Laboratory of Digital Manufacturing Equipment and Technology, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
[c] Hubei Key Laboratory of Mechanical Transmission and Manufacturing Engineering, School of Machinery and Automation, Wuhan University of Science and Technology, Wuhan 430081, China

ARTICLE INFO

*Article history:*
Received 13 June 2017
Accepted 14 September 2017
Available online 23 September 2017

*Keywords:*
Transient response
Multiple viscoelastic damping
Precise time integration
Reduction methods
Modal projection

ABSTRACT

Large and complex engineering systems are usually assembled by subcomponents with different energy dissipation levels. Therefore, these systems often contain multiple damping models, which may lead to great difficulties in analyzing efficiently. In this paper, an efficient modal projection-based reduction method, which accounts for transient dynamic responses of structural system with multiple damping models, is proposed in the framework of a modified precise integration method. Two robust modal reduction bases, namely multi-model method (MM) and modal strain energy by first-order correction method (MSEC), are introduced to reduce the order of the original system. Based on the reduced system and a general damping model (GDM), a reduced state-space formalism for the structural system with multiple damping models is developed. Finally, the transient dynamic responses are derived using a modified precise integration method on the reduced stage. The numerical stability, accuracy and complexity are discussed. Two numerical examples are illustrated to assess the performances of the computational accuracy and efficiency. The results indicate that the proposed method is more efficient than other methods and most suitable for large-scale problems with rather good accuracy.


## 1. Introduction

Advanced composite materials and complicated structural systems are extensively applied to aerospace, ship, civil and automobile fields, leading to various subcomponents with significantly different energy dissipation levels. Therefore, modern engineering systems often contain multiple damping models. In order to accurately express the dissipative forces, these energy dissipation characteristics should be described by different viscous and viscoelastic damping models. The damping model which depends on the past history of motion via convolution integrals over some kernel functions has been considered to be the most general damping models in linear system [1]. The convolution damping model is suitable for systems varying with frequency. In theory, the kernel functions can be any mathematic model on condition that it makes the energy dissipation functional nonnegative. However, any modification of the kernel function may bring about some difficulties in analyzing the corresponding structural system. At present, the studies of the viscoelastic damping systems are mainly focused on efficiently solving the eigensolutions [2–8] and the harmonic dynamic responses [9–11]. In practice, the structural systems are often subjected to sudden loadings (such as impact, sine and seismic loadings), but only a handful of studies are applicable to the calculation of transient dynamic responses for viscoelastic damping systems, especially for the systems with multiple damping models.

In the last two decades, some authors have considered the dynamic responses of viscoelastic damping systems with exponential type relaxation kernels [1]. Menon and Tang [12] presented a state-space formulation of the viscoelastic systems and solved the dynamic response by using standard state-space packages. In order to improve the computational efficiency, Adhikari and Wagner [13] developed a direct time-domain integration method based on an extended state-space formulation for exponentially damped

* Corresponding author at: Key Laboratory of Metallurgical Equipment and Control Technology, Ministry of Education, School of Machinery and Automation, Wuhan University of Science and Technology, Wuhan 430081, China.

*E-mail addresses:* dingzhe@wust.edu.cn, dingzhe@hust.edu.cn (Z. Ding), li-li_em@hust.edu.cn (L. Li), kongjianyi@wust.edu.cn (J. Kong), qinli429@wust.edu.cn (L. Qin).

http://dx.doi.org/10.1016/j.compstruc.2017.09.004
0045-7949/

linear systems [2]. To further enhance the efficiency, Cortés et al. [14] transformed the motion equation into a differential equation by the Laplace transformation and calculated the dynamic responses by an implicit integration method. The method employs no internal variables, but only applicable when the order of the exponential kernel function is no higher than two. Pan and Wang [15] utilized standard Discrete Fourier Transform procedure together with Fast Fourier Transform algorithms to compute the dynamic responses of the exponentially damped systems with non-zero initial conditions. For other viscoelastic damping models, Palmeri and Muscolino [16] considered a state-space representation of generalized Maxwell's model and calculated the transient dynamic response by using Laguerre's polynomial approximation technique. Muravyov [17] also studied the dynamic responses of the viscoelastic systems in time-domain with some frequency-dependent damping models.

For structural systems with multiple damping models, Failla et al. [18] addressed the harmonic responses of bars with an arbitrary number of dampers. Burlon et al. [19] investigated the frequency response of Euler-Bernoulli beams carrying multiple viscoelastic damping models. Liu [20] and Puthanpurayil et al. [21] developed two implicit integration methods based on Newmark integration for viscoelastically damped systems successively. These two methods are available to all possible damping kernel functions but suffered from high computational burden. Then, Liu [22] proposed an explicit integration method based on central difference formula of acceleration. The method is proved to be more efficient than the implicit method, but the stable condition is unclear. Li and Hu [23] introduced a unified way to express the commonly used viscoelastic damping models and derived its corresponding state-space formulation. Later on, Ding et al. [24] developed a state-space based time integration scheme for the transient dynamic response of structural systems with multiple viscoelastic damping models. However, the velocity responses may have some significant computational errors in special cases due to the approximations of the revelent vectors. Recently, Ding et al. [25] presented a modified precise integration method with Gauss-Legendre quadrature to compute the transient dynamic response of multiple damping systems. The method shows a superior advantage on computational accuracy, but may have some restrictions on efficiently solving large-scale problems.

The usual way of treating viscoelastic effects in structural systems for transient dynamic response is to introduce extra dissipation coordinates or internal variables [26]. This transformation could lead to high computational cost in the case of large-scale structures, especially those with multiple damping models. Model reduction method, which allows to decrease the size of problem dramatically, is an ideal choice to improve the computational efficiency for transient response analysis of structures with multiple damping models. Tang et al. [27] developed a dynamic model-reduction method for rods with frequency-dependent damping. Park et al. [28] proposed an internal balancing model reduction method to remove the internal variables accounting for viscoelastic properties in finite element model. Boumediene et al. [29] presented a reduction method for damped viscoelastic sandwich structures of free vibration based on the high order Newton algorithm. Then, Boumediene et al. [30] derived an asymptotic numerical algorithm based reduction method to determine the forced harmonic response of viscoelastic sandwich structures. Besides, some component mode synthesis (CMS) methods are also presented for viscoelastic damped structures [31–33]. Recently, Rouleau et al. [34] compared several model reduction techniques based on modal projection for structures with frequency-dependent damping and drawn some instructive conclusions. However, few attentions are paid to the transient dynamic response for viscoelastic systems based on reduction methods. Zghal et al. [35] compared some reduction methods for viscoelastic sandwich structures and computed the dynamic response in the time domain by using Newmark's integration method [36].

Direct time integration method is one of the most popular methods to capture the transient dynamic response of structural systems in time domain, such as the Newmark method, the Wilson-$\theta$ method and the central difference formula, etc. Among these methods, the precise integration method (PIM) [37], with rather high accuracy and efficiency, has been widely used to calculate the transient dynamic response of structural systems in many fields [38–40]. Wang and Au [41] proved the PIM method to be conditionally stable and the stable conditions were easy to be satisfied. However, if the PIM method is applied to the viscoelastic damping systems, large computational cost and storage space may be needed for large-scale structures. Therefore, a modified PIM method combined with a robust model reduction algorithm may be a suitable candidate for the transient dynamic analysis of structural system with multiple damping models by considering both computational accuracy and efficiency.

The aim of this work is to develop a new time integration method for structural systems involving multiple damping models based on the modal projection-based reduction method with higher efficiency. Firstly, two effective modal-based reduction methods for viscoelastic damping systems are introduced. Secondly, the original system matrices are condensed by using the reduction methods. Then, the transient dynamic responses of the structural systems with multiple damping models are deduced based on a modified PIM method and the reduced system matrices. The computational accuracy, efficiency and complexity are discussed and compared. The proposed method is proved to be of high efficiency and the stable condition is easy to be satisfied.

The paper is organized as follows: in Section 2, the problem is formulated by presenting some theoretical backgrounds, reviewing some kernel functions and introducing two modal-based reduction methods for viscoelastic damping systems. In Section 3, the modal projection-based reduction method for transient dynamic responses of structural viscoelastic systems is derived and some computational considerations, including stability, accuracy and computational complexity, are investigated. In Section 4, two numerical examples are presented and discussed to assess the performances of the proposed methods. Finally, some important conclusions will be summarized in Section 5.

## 2. Theoretical backgrounds and problem formulations

### 2.1. Governing equation of structural systems with multiple viscoelastic damping models

The equations of motion of an $N$ degree-of-freedom (DOF) linear discrete system with multiple viscoelastic damping models, which depend on the past history of motion via convolution integrals over some suitable kernel functions, can be expressed in time domain by [23–25]

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}_0\dot{\mathbf{x}}(t) + \sum_{k=1}^{n}\mathbf{C}_k\int_0^t g_k(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

with initial conditions

$$\mathbf{x}(t=0) = \mathbf{x}_0 \in \mathbb{R}^N, \quad \dot{\mathbf{x}}(t=0) = \dot{\mathbf{x}}_0 \in \mathbb{R}^N \tag{2}$$

Here $t$ denotes time and $\tau$ is retardation time. $\mathbf{M}, \mathbf{K} \in \mathbb{R}^{N\times N}$ are mass and stiffness matrix, respectively. $\mathbf{x}(t), \dot{\mathbf{x}}(t), \ddot{\mathbf{x}}(t) \in \mathbb{R}^N$ are displacement vector and its first and second time derivatives. $\mathbf{f}(t) \in \mathbb{R}^N$ is forcing vector. $\mathbf{C}_0 \in \mathbb{R}^{N\times N}$ is frequency-independent damping matrix which takes into account the viscous damping