Computers and Structures 194 (2018) 74–85

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Simultaneous topology optimization of supporting structure and loci of isolators in an active vibration isolation system

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ARTICLE INFO

Article history:
Received 19 April 2017
Accepted 16 September 2017

Keywords:
Active vibration isolation
Supporting structure
Loci of isolators
Multi-level optimization

ABSTRACT

We developed a new multi-objective and multi-level optimization method to design an active vibration isolation system. Both the layout of the continuum (i.e. the supporting structure) and the loci of the isolators are designed using topology optimization technique in a unified formulation for the first time. The static, dynamic and vibration-isolation characteristics are taken into account simultaneously in the present model. Due to their different roles in the system it may be appropriate and advantageous to treat the design of the continuum layout and isolator loci as different sub-problems with different objectives in separate stages. The multi-level optimization technique, where the optimization of the supporting structure and the isolator loci are incorporated into a closed-loop, is proposed and implemented so that the interactions between these two sub-problems can be fully taken into account. Numerical results demonstrate the validity of the proposed design cycle. Comparisons show that the overall static, dynamic and vibration-isolation performance of the optimized system outperforms the ones designed by traditional methods.

1. Introduction

Topology optimization of structures considering dynamic characteristics has drawn the interests of many researchers since 1990s. Generally speaking, the dynamics related topology optimization is much more intricate than the one concerning static characteristics only. Some typical obstacles of the former include the so-called localized modes [1], the possible non-differentiability of the eigenfrequencies [2–4] and the higher computational cost when compared to static analysis [5,6].

Basically, existing literatures on the dynamics related topology optimization can be categorized into three groups:

- In the first group, the eigenfrequencies of structures are the topic, either the eigenfrequencies of some fixed orders [2,4,7–10] or the eigenfrequency gap [3,11,12] is taken as the objective function. The goal is to maximize the eigenfrequency or the eigenfrequency gap with limited material usage.
- In the second group, the steady-state frequency response of structures is of interests and the goal is to minimize the structural response under the harmonic excitation of some fixed frequencies or among some frequency ranges [13]. The objective function may be taken as the dynamic compliance [14,15], the (weighted) amplitude of the displacements [16–18] of some specified nodes, etc.
- In the third group, the transient response of structures is concerned and the aim is to minimize the structural response at some fixed time or in a time range under transient time-domain excitation [19–22]. For an insight into this group, we recommend the review paper [23] and the references therein.

In the simplest vibration isolation theory of single-dof system, the active vibration isolation system is composed of the vibration source (the machines), the isolators and the ground. While in engineering practice the supporting structure is often included in the system to link different machines and to facilitate the installation and maintenance.

As shown in Fig. 1, in this paper we focus on the design of an active vibration isolation system where both the dynamic and the static characteristics of the system, as well as the vibration isolation performance should be considered simultaneously.

The continuum (i.e. the supporting structure) will be designed using traditional density-based topology optimization method. When optimizing the loci of the isolators, the intuitionistic idea of taking the coordinates of the isolators as parameters and doing parametric optimization to find the best coordinates would be very

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https://doi.org/10.1016/j.compstruc.2017.09.006
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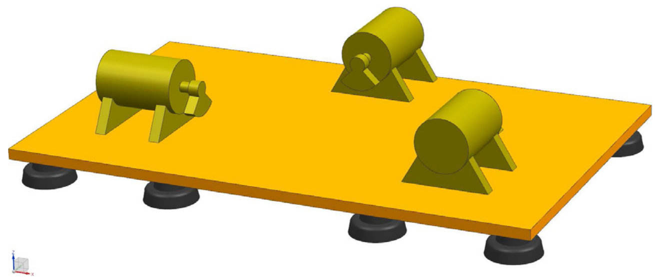

Fig. 1. An active vibration isolation system adapted from engineering practice. The orange one is the supporting structure. The strong oliver ones (generators) are the machines and the vibration source in this system. The black ones are the rubber isolators. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

inefficient. FEA programs in solid mechanics are based on Lagrangian mesh, so modifying the coordinates of the isolators would directly lead to the time-consuming remesh process. Another idea is to choose the best loci among some predefined locations. But this requires to solve the 0–1 programming problem together with the topology optimization of the continuum so as to be inefficient as well.

In this paper, we presents the topological optimization model in which both the layout of the continuum and the loci of the isolators are described by continuous topological variables. This way, the design of the loci of the isolators can be accomplished by solving the normal optimization problem with only continuous variables.

The roles of the supporting structure and the isolators are quite different, so they should be designed under different principles:

- The requirements for the supporting structure is that it should have enough static and dynamic stiffness. So when designing the supporting structure, the objectives may be chosen as the dynamic and static compliance. In some industrial applications, such as the isolation systems in the engine room and motor room of a ship [24–26], the design of the supporting structure is a very important topic.
- The isolators are the backbone to improve the vibration isolation performance and to ensure the safety of the machine(s). So when designing the isolators, the target should be to minimize the forces transmitted into the ground and to ensure the safety of the machines as well. Here the safety of the machines can be measured by the displacements at the machines [27].

Considering the essential difference between the isolators (which are modelled as discrete spring elements) and the supporting structure (which is modelled as continuum shell elements), it is both appropriate and advantageous to decompose the above principles and to design the supporting structure and the isolators in separate stages by using the strategy of divide-and-conquer. However, the abovementioned principles are actually mutually affected by each other. The change in the topology of the structure will surely influence the optimal locations of the isolators, and vice versa. So the design of the supporting structure and the isolators should be included in a closed-loop to fully take into account the interactions between each other. This way, the whole system may be designed by the multi-level optimization method. Considering the interactions between different parts is indeed the core of the divide-and-conquer strategy.

The techniques of decomposition and multi-level optimization have been discussed extensively in the literature. Readers interested in this topic can refer to the Chapter 11 of Haftka's monograph [28] and the references therein. Optimizing the loci of the isolators together with the topology of continuum structures in an active vibration isolation system seems to be new in the research area of topology optimization and has not been investigated by others. But optimizing the supports in truss structures [29–33] and continuum structures [34,35] can be found in the literature.

The remaining parts are arranged as follows. In Section 2 the basic FEA procedure of the steady-state dynamics including the SIMP interpolation scheme is presented. In Section 3 the objective functions for designing the supporting structure and the isolators are formulated based on the FEA procedure shown in Section 2. Using these objective functions, in Section 4 the supporting structure and the isolators are designed successively following some specific work flow. In Section 5 some numerical results are given, then the validity of the proposed methods and the superiority of the optimized system are verified. Some discussions on the obtained optimized design are given in Section 6. The conclusions are made in Section 7. In Appendix A the sensitivity information of the objective functions formulated in Section 3 are presented.

2. Basic FEA procedure and SIMP interpolation

The motion equation of the structural dynamics is given by

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \tag{1}$$

where $\boldsymbol{M}, \boldsymbol{K} \in \mathbb{R}^{n_{\mathrm{dof}} \times n_{\mathrm{dof}}}$ are the global mass matrix and stiffness matrix, respectively; $\boldsymbol{f}, \boldsymbol{u} \in \mathbb{R}^{n_{\mathrm{dof}}}$ are the global load and displacement vector, respectively; $\mathbb{R}$ is the real subspace; n_{dof} is the number of dofs in the FEA model.

The global stiffness matrix is composed of the stiffness matrix of continuum elements (i.e. the shell elements) and the spring elements, while the global mass matrix is composed of the mass matrix of continuum elements and the point-mass elements:

$$\boldsymbol{K} = \boldsymbol{K}^{\mathrm{sh}} + \boldsymbol{K}^{\mathrm{sp}} \tag{2a}$$

$$\boldsymbol{M} = \boldsymbol{M}^{\mathrm{sh}} + \boldsymbol{M}^{\mathrm{pt}} \tag{2b}$$

where *sh*, *sp* and *pt* are short for shell elements, spring elements and point-mass elements, respectively. Further $\boldsymbol{K}^{\mathrm{sh}}, \boldsymbol{M}^{\mathrm{sh}}, \boldsymbol{K}^{\mathrm{sp}}$ and $\boldsymbol{M}^{\mathrm{pt}}$ come from the finite elements assembly:

$$\boldsymbol{K}^{\mathrm{sh}} = \sum_{i=1}^{n_{\mathrm{sh}}} \overline{\boldsymbol{K}}_i^{\mathrm{sh,e}} \tag{3a}$$

$$\boldsymbol{M}^{\mathrm{sh}} = \sum_{i=1}^{n_{\mathrm{sh}}} \overline{\boldsymbol{M}}_i^{\mathrm{sh,e}} \tag{3b}$$

$$\boldsymbol{K}^{\mathrm{sp}} = \sum_{j=1}^{n_{\mathrm{sp}}} \overline{\boldsymbol{K}}_j^{\mathrm{sp,e}} \tag{3c}$$

$$\boldsymbol{M}^{\mathrm{pt}} = \sum_{k=1}^{n_{\mathrm{pt}}} \boldsymbol{M}_k^{\mathrm{pt,e}} \tag{3d}$$

where $n_{\mathrm{sh}}, n_{\mathrm{sp}}, n_{\mathrm{pt}}$ are the number of shell elements, spring elements and point-mass elements, respectively; $\overline{\boldsymbol{K}}_i^{\mathrm{sh,e}}$ and $\overline{\boldsymbol{M}}_i^{\mathrm{sh,e}}$ are the actual stiffness matrix and mass matrix for *i*-th shell element, they are related to the nominal stiffness and mass matrix through the famous SIMP interpolation scheme [15,16]:

$$\overline{\boldsymbol{K}}_i^{\mathrm{sh,e}} = x_i^3 \boldsymbol{K}_i^{\mathrm{sh,e}} \tag{4a}$$

$$\overline{\boldsymbol{M}}_i^{\mathrm{sh,e}} = x_i^3 \boldsymbol{M}_i^{\mathrm{sh,e}} \tag{4b}$$

where $x_i \in [0, 1]$ is the topological variable for *i*-th shell element.

Similarly, $\overline{\boldsymbol{K}}_j^{\mathrm{sp,e}}$ is the actual stiffness matrix for *j*-th spring element, $\boldsymbol{M}_k^{\mathrm{pt,e}}$ is the mass matrix for *k*-th point-mass element. $\overline{\boldsymbol{K}}_j^{\mathrm{sp,e}}$ is related to the nominal elemental stiffness matrix through: