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A novel sub-step composite implicit time integration scheme for structural dynamics

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ABSTRACT

In this paper, a novel sub-step composite implicit time integration scheme is presented for solving the problems in structural dynamics. The proposed scheme possesses desirable stability and accuracy. With appropriate algorithmic parameter value, the scheme can attain controllable amplitude decay and period elongation. Effectiveness of the proposed scheme is tested in some example solutions by comparing with other well-known implicit schemes. Theoretical analysis and numerical simulations demonstrate that the proposed scheme possesses high computation efficiency as well as desirable numerical dissipation characteristics.

1. Introduction

A large amount of research has been conducted to explore effective time integration schemes for linear and nonlinear analysis of structures [1–9]. For explicit methods, the central difference method is commonly used. As for implicit methods, there are a large number of methods presented. Implicit methods are preferred choice for some specific problems due to its advantages over explicit methods. The trapezoidal rule and Generalized-α method are most commonly used implicit methods [9,10].

For linear analysis, the trapezoidal rule is unconditionally stable, second-order accurate, and shows no amplitude decay (*AD*) and acceptable period elongation (*PE*), however, this scheme is unstable in nonlinear analysis where conditions of momentum and energy conservation are not satisfied [9]. Another drawback of the trapezoidal rule is in that it gives no numerical dissipation which is significant for finite element analysis of structures. One approach to overcome this drawback is to introduce some damping by introducing some parameters into a time integration method, and the Generalized-α method is the most representative case [10], however, to acquire acceptable accuracy, its parameters need to be cautiously selected according to the characteristics of the problem solved. Bathe and Baig [11] presented a composite implicit time integration procedure which is found to be effective where the trapezoidal rule fails to produce a stable solution. This composite implicit scheme is then successfully applied for certain nonlinear dynamic analysis [3]. Recently, to improve numerical damping characteristics, we proposed a new family of time integration methods where the selection of parameters is flexible [12,13], however, new methods entails a large amount of matrix calculation, which hinders its practical application.

In this paper, inspired by the work of Bathe and Baig [11], a new implicit scheme is presented with the introduction of one free parameter. In the three sub-step composite scheme by Bathe and Baig [11], the trapezoidal rule is used for the first and second sub-steps, and the Houbolt method is used for the third sub-step. As for the proposed scheme, the trapezoidal rule and the Houbolt method are also used for the first and third sub-steps respectively, but, for the second sub-step, the Euler backward method is adopted. New scheme is expected to be desirable in calculation accuracy and numerical dissipation characteristics. In the following, we first give the basic equations of the scheme, then present some basic properties of the time integration method, finally, solved some representative examples to confirm some important and valuable properties of the scheme.

2. The time integration scheme

2.1. Standard formulations

Considering linear analysis, the governing finite element equation to be solved are

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$$\mathbf{M\ddot{U}} + \mathbf{D\dot{U}} + \mathbf{KU} = \mathbf{R} \tag{1}$$

where $\mathbf{U}$ is the vector of nodal displacements (including rotations). $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ are, respectively, the first and second derivatives of $\mathbf{U}$ in terms of time t. $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively. $\mathbf{R}$ is the applied loads vector. For nonlinear problems, $\mathbf{D}$ and $\mathbf{K}$ can be obtained by $\mathbf{D} = \frac{\partial \mathbf{R}_d}{\partial \dot{\mathbf{U}}}$ and $\mathbf{K} = \frac{\partial \mathbf{R}_s}{\partial \mathbf{U}}$, where $\mathbf{R}_d$ and $\mathbf{R}_s$ are the nodal damping force and the elastic force corresponding to the element internal stresses, respectively.

In this proposed scheme, the complete time step Δt is divided into three different sub-steps as $p\Delta t$, $(1-2p)\Delta t$ and $p\Delta t$, where $0 < p < 0.5$. For the first sub-step the trapezoidal rule is employed, for the second sub-step 3-point Euler backward method is adopted and the Houbolt method is used for the third sub-step. For any MDOF system [11], they are, respectively, formulated as

$$^{t+p\Delta t}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \frac{1}{2}p\Delta t({}^{t}\ddot{\mathbf{U}} + {}^{t+p\Delta t}\ddot{\mathbf{U}}) \tag{2}$$

$$^{t+p\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + \frac{1}{2}p\Delta t({}^{t}\dot{\mathbf{U}} + {}^{t+p\Delta t}\dot{\mathbf{U}}) \tag{3}$$

$$(\Delta t) \cdot {}^{t+(1-p)\Delta t}\dot{\mathbf{U}} = c_1{}^{t}\mathbf{U} + c_2{}^{t+p\Delta t}\mathbf{U} + c_3{}^{t+(1-p)\Delta t}\mathbf{U} \tag{4}$$

$$(\Delta t) \cdot {}^{t+(1-p)\Delta t}\ddot{\mathbf{U}} = c_1{}^{t}\dot{\mathbf{U}} + c_2{}^{t+p\Delta t}\dot{\mathbf{U}} + c_3{}^{t+(1-p)\Delta t}\dot{\mathbf{U}} \tag{5}$$

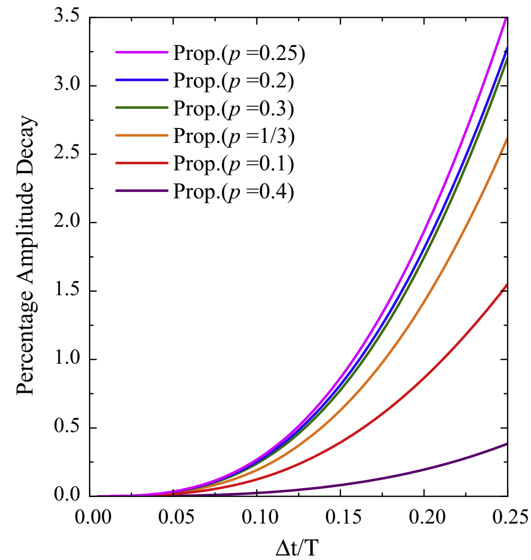

Fig. 3. Percentage Amplitude Decay (AD) of the proposed scheme for various p.

$$(\Delta t) \cdot {}^{t+\Delta t}\dot{\mathbf{U}} = d_1{}^{t}\mathbf{U} + d_2{}^{t+p\Delta t}\mathbf{U} + d_3{}^{t+(1-p)\Delta t}\mathbf{U} + d_4{}^{t+\Delta t}\mathbf{U} \tag{6}$$

$$(\Delta t) \cdot {}^{t+\Delta t}\ddot{\mathbf{U}} = d_1{}^{t}\dot{\mathbf{U}} + d_2{}^{t+p\Delta t}\dot{\mathbf{U}} + d_3{}^{t+(1-p)\Delta t}\dot{\mathbf{U}} + d_4{}^{t+\Delta t}\dot{\mathbf{U}} \tag{7}$$

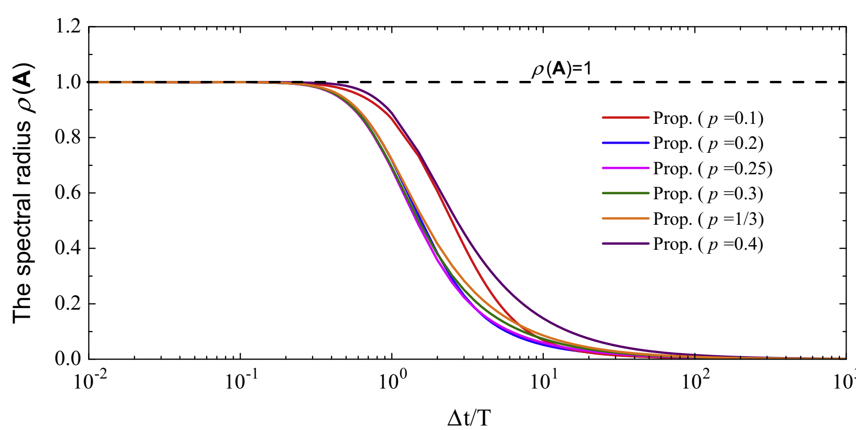

Fig. 1. Spectral radii $\rho(\mathbf{A})$ of the proposed method, case $\xi = 0$, for different values of p.

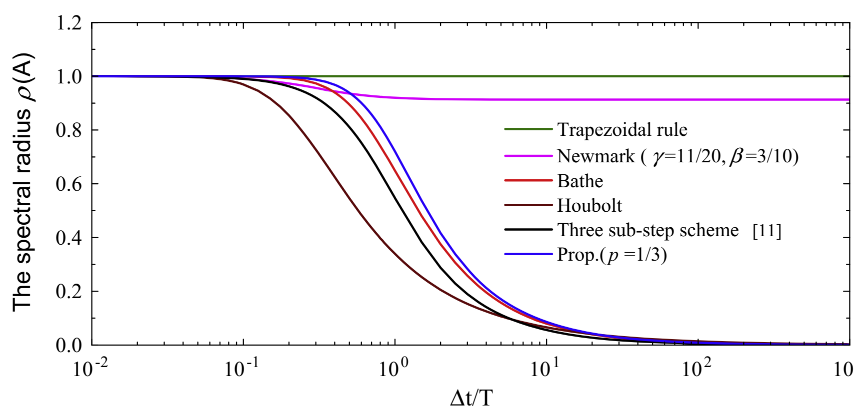

Fig. 2. Spectral radii of approximation operators, case $\xi = 0$, for various schemes.