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The combination of the boundary element method and the numerical manifold method for potential problems

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ABSTRACT

In this study, a boundary element method coupled with numerical manifold method is developed for solving potential problems in two dimension. This approach combines an equivalent variational form of a boundary integral equation with the finite cover approximations for generating the trial and test functions of the variational formulation. This method exploits the reduced dimensionality advantages of the BEM, and is especially suited for the problems with an unbounded domain. Since the local cover function can be chosen in the covers arbitrarily, the method provides flexibility to use different cover functions for different covers and increases the solution accuracy without any local mesh refinement, and the *p*-adaptive analysis can also be performed conveniently. The validity and efficiency of the present method are demonstrated by some numerical examples of potential problems.

1. Introduction

The numerical manifold method (NMM) [1,2] is a novel computational approach which can analyze general continuous and discontinuous problems in a unified way. The main feature of this method is that there are two separated and independent cover system, i.e. mathematical covers and physical covers. Based on the finite cover approximation theory, the NMM combines the finite element method (FEM) and the discontinuous deformation analysis (DDA) so that this method is more suitable than other numerical methods for problems with discontinuous and moving boundaries, e.g. crack propagation problems [3–7], free surface flow problems [8–11], and so on. In recent years, this method has made great achievements in many fields.

Compared with the FEM, the NMM can perform the *p*-adaptive analysis without limitation. Chen et al. [12] derived detailed formulations of the high-order NMM. Based on tetrahedral meshes, Jiang et al. [13] extended this method to 3D. Zheng et al. [14] constructed the numerical manifold space of the Hermitian form to solve the Kirchhoff's thin plate problems, and made some earliest developed elements in finite element history, e.g. Zienkiewicz's plate element, to regain their vigor. Wong and Wu [15] adopted the displacement-dependent cohesion removal method to overcome the limitation of the original NMM associated with an improper removal of the interface cohesion of the discontinuities, and investigated the progressive failure in rock slopes. Through modifying the finite element partition of unity (PU) into the flat-top PU, the linear dependence problem involved in finite element PU-based high-order polynomial approximation was successfully alleviated [16]. By coupling the NMM with non-uniform rational B-splines (NURBS) and T-splines, the numerical manifold method based on isogeometric analysis was proposed by Zhang et al. [17], and the local refinement technique using T-splines reduced the number of degrees of freedom while maintaining calculation accuracy at the same time. In order to avoid completely the linear dependence problem in theory, Fan et al. developed two approaches in NMM, one employed the nine-node triangular meshes which the high-order PU function was employed [18], and the other added the derivative degrees of freedom with physical meaning to original displacement degrees of freedom [19]. In the conventional NMM, the false volume expansion and other issues exist in solving large deformation and large rotation problems. In view of this consideration, the S-R (strain-rotation) decomposition and the generalized-α method were introduced, Fan et al. [20] established the S-R-D based NMM, improved the performances of this approach in the analysis of dynamic large deformations, and demonstrated the superiority and potential of the NMM.

The above numerical manifold methods can be considered as the generalization of the classical FEM. Meanwhile, the boundary element method (BEM) [21] is also an attractive numerical method for solving a wide variety of computational engineering and science problems as it

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can reduce the dimensionality of the original problem by one. Especially for exterior problems, the FEM requires discretization of the entire exterior, whereas with the BEM only the surface needs to be discretized. The BEM has been successfully used in fields of elasticity [21], geomechanics [22,23], fracture mechanics [24], and so on. In order to solve large-scale problems, Liu et al. [25] introduced the fast multipole method into the BEM. In concept of isogeometric analysis, Simpson et al. [26] proposed an isogeometric boundary element method (IGABEM), in which the NURBS basis permitted exact representation of commonly used geometries including circular arcs, and a meshing process was no longer required. Combing the meshless idea with the boundary integral equations, some boundary type meshless methods were also developed, e.g. the boundary node method (BNM) [27], the local boundary integral equation (LBIE) method [28], the hybrid boundary node method (HBNM) [29], the boundary element-free method (BEFM) [30,31], the Galerkin boundary node method (GBNM) [32–34], and so on.

In this study, a boundary element method coupled with numerical manifold method is developed for solving potential problems in two dimension. The finite cover approximation is implemented to construct the trial and test functions of the variational form so that the *p*-adaptive analysis can be performed conveniently. This method also exploits the reduced dimensionality advantages of the BEM, and is especially suited for the problems with an unbounded domain. Moreover, the 'stiffness' matrices are symmetric, which makes this approach computationally efficient and provides advantage for coupling it with finite element method.

The discussion of this method are arranged as follows: Section 2 introduces some basic theory in the NMM. In Section 3, a detailed numerical implementation of the proposed method is described for solving potential problems. Section 4 gives some numerical examples. Finally, the paper will be ending with conclusions in Section 5.

2. The NMM approximation scheme

2.1. Basic concepts of the NMM

The NMM approximation is based on three basic concepts, i.e. the mathematical cover (MC), the physical cover (PC) and the manifold element. The mathematical covers define only the fine or rough approximations, so they are chosen by users, and consist of finite overlapping covers which occupy the whole domain. Usually, the conventional meshes and regions can be transferred to mathematical covers. On each mathematical cover, a partition of unity function is defined. While the physical cover system is formed by intersecting mathematical covers and the physical features, such as the boundaries of domain, the discontinuities, the material interfaces and so on. On each physical cover, a local function is defined. The common area of several physical covers forms a manifold element. The NMM employs the partition of unity functions to paste all the local functions together to give a global approximation over each manifold element. Further details of geometrical aspects of the NMM were deliberately elucidated in Refs. [1,5].

2.2. The NMM approximation technique

The NMM approximations are used for constructing the trial and test functions on boundary. The cover weight functions are practically equivalent to the shape functions in the boundary element analysis. The local cover function can be of various forms such as a constant basis function, a linear basis function, and a higher-order polynomial basis function.

Let Γ be a smooth, simple closed curve in the plane, and let Ω and Ω' denote its interior and exterior respectively. Usually, the mathematical cover and the physical cover are generally independent. In Fig. 1, only the boundary of domain is intersected with mathematical covers,

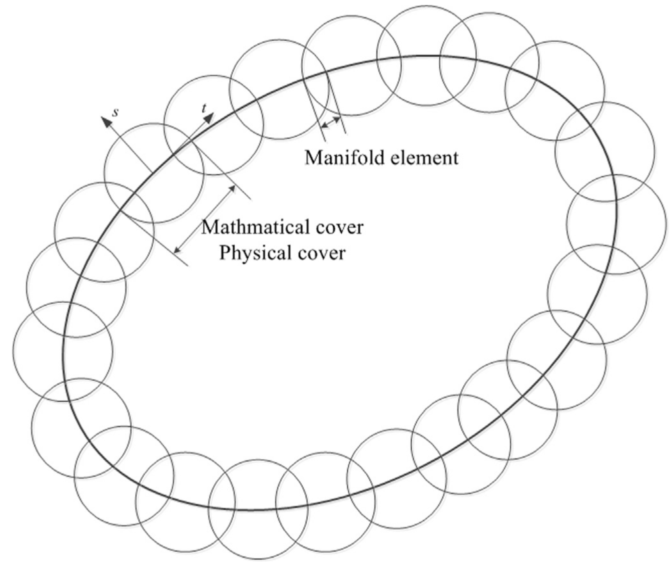

Fig. 1. Problem domain, mathematical cover, physical cover and manifold element.

so the physical cover overlaps with the mathematical cover. We use N covers, i.e., U_i, $i = 1, 2, \ldots, N$, to occupy the whole boundary Γ(see Fig. 1). The local cover function v_i of any cover U_i can be constructed by a linear combination of mutually independent functions $f_{i,j}$ of a given order m and the constant coefficients $d_{i,j}$, and can be written in a matrix form as

$$v_i(\mathbf{x}) = \sum_{j=1}^{m} f_{i,j}\, d_{i,j} = \mathbf{f_i d_i} \tag{1}$$

where $\mathbf{f_i} = [1, x_1, x_2, x_1^2, x_1x_2, x_2^2, \cdots]$ is a vector of the polynomial basis; $\mathbf{d_i} = [d_{i,1}, d_{i,2}, \ldots, d_{i,m}]^T$ is called the local degree of the freedom vector.

Assume that any manifold element e presents the common part of several overlapped covers $U_{e(i)}(i = 1, 2, \ldots, q)$, and the function $w_{e(i)}(\mathbf{x})$ is the weight function corresponding to the cover $U_{e(i)}$. The displacement function of the manifold element e can be approximated as

$$v(\mathbf{x}) = \sum_{i=1}^{q} w_{e(i)}(\mathbf{x}) v_{e(i)}(\mathbf{x}) = \sum_{i=1}^{q} \sum_{j=1}^{m} w_{e(i)}(\mathbf{x}) f_{e(i),j}(\mathbf{x}) d_{e(i),j} = \sum_{i=1}^{q} T_i(\mathbf{x}) d_i \tag{2}$$

where $T_i(\mathbf{x}) = w_{e(i)}(\mathbf{x})\mathbf{f}_{e(i)}$ is a row vector of order m, and d_i is column vector of order m.

3. Boundary element method coupled with NMM

Consider the interior and exterior Dirichlet problems

$$\begin{cases} \nabla^2 u = 0 \quad \text{in } \Omega \text{ or } \Omega' \\ u = \bar{u} \quad on\ \Gamma \end{cases} \tag{3}$$

and Neumann problems

$$\begin{cases} \nabla^2 u = 0 \quad \text{in } \Omega \text{ or } \Omega' \\ q \equiv \partial u/\partial n = \bar{q} \quad on\ \Gamma \end{cases} \tag{4}$$

where $\bar{u}$ and $\bar{q}$ are the prescribed functions; and n is the unit outward normal.

The solution of Dirichlet problem, i.e. Eq. (3) can be represented by a double layer potential [33]

$$\begin{cases} u(\mathbf{x}) = -\int_\Gamma v(\mathbf{y}) \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial \mathbf{n_y}} dS_y \\ q(\mathbf{x}) = -\int_\Gamma v(\mathbf{y}) \frac{\partial u^*(\mathbf{x},\mathbf{y})}{\partial \mathbf{n_y} \partial \mathbf{n_x}} dS_y \end{cases} \tag{5}$$

and Neumann problems, i.e. Eq. (4) can be solved via a simple layer potential [33]