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Boundary element analysis of 3D cracks in anisotropic thermomagnetoelectroelastic solids

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ABSTRACT

The paper presents a general boundary element approach for analysis of 3D cracks in anisotropic thermomagnetoelectroelastic solids. Dual boundary integral equations are derived, which kernels are explicitly written. These equations do not contain volume integrals in the absence of distributed body heat and extended body forces, which is advantageous comparing to the existing approaches. The issues on the boundary element solution of these equations are discussed in details. The efficient numerical evaluation of kernels based on the trapezoid rule is proposed. Modified Kutt's quadrature with Chebyshev nodes is derived for integration of singular and hypersingular integrals. Nonlinear polynomial mappings are adopted for smoothing the integrand at the crack front, which is advantageous for accurate evaluation of field intensity factors. Special shape functions are introduced, which account for a square-root singularity of extended stress and heat flux at the crack front. The issues on numerical determination of field intensity factors are discussed. Several numerical examples are presented, which show the efficiency (low computational time and high precision) of the proposed boundary element formulation.

1. Introduction

Thermomagnetoelectroelastic (TMEE) materials are used in the wide range of modern precise devices. Those are smart structures (pyroelectrics, pyromagnetics and composite materials containing both phases), which can convert different fields, and serve as sensors, actuators or even complex micro-electro-mechanical systems. The rapid development of modern multi-field materials and micro-electro-mechanical technologies raises increasing attention to their modeling and simulation. Particular interest is focused on the issues of fracture mechanics of TMEE materials [1]. Since TMEE materials are anisotropic by nature, their analysis is more complicated than those of isotropic materials.

The boundary element method (BEM) is widely applied in the linear fracture mechanics [2–4], since it allows accurate evaluation of field intensity factors at the crack front and requires only boundary mesh. Various boundary element techniques were proposed for 3D fracture mechanics analysis. Mi and Aliabadi [3] derived a 3D dual BEM for analysis of 3D cracks in isotropic linear elastic solids. Saez et al. [5] presented a boundary element formulation for crack analysis in transversely isotropic solids. Pan and Yuan [6] developed a single-domain BEM for 3D fracture mechanics analysis in generally anisotropic solids. Rungamornrat and Mear [7] derived a symmetric Galerkin BEM for analysis of cracks in 3D anisotropic media. These approaches are closely related to the techniques of anisotropic Green's functions evaluation [8–10], since the latter significantly influence efficiency and accuracy of the BEM.

Nevertheless, coupling of different fields in the solid's material significantly complicates boundary element formulations. Many researches address the issues of thermal expansion influence on fracture parameters. In instance, Dell'erba et al. [11] developed a dual BEM for 3D thermoelastic crack problems. Mukherjee et al. [12] derived regularized hypersingular boundary integral equations for isotropic thermoelastic fracture mechanics.

A number of works address piezoelectric, piezomagnetic and magnetoelectroelastic coupling. Rungamornrat and Mear [13] and Rungamornrat et al. [14] derived a symmetric Galerkin BEM for 3D fracture mechanics analysis of piezoelectric solids. Zhao et al. [15] presented the extended discontinuity boundary integral equation method for vertical cracks in magnetoelectroelastic medium. Muñoz-Reja et al. [16] presented the 3D BEM for fracture mechanics analysis of anisotropic magnetoelectroelastic materials.

However, to this end there is no general 3D BEM for analysis of 3D cracks in anisotropic medium, which couples both thermal and

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magneto-electro-mechanical fields. Several papers [17–19] address only the particular problems for a penny-shaped crack or two parallel concentric circular cracks in a thermopiezoelectric medium. No works addressing TMEE medium containing 3D cracks of arbitrary shape were found in scientific literature.

Besides, there is no single and efficient approach for evaluation of anisotropic kernel functions, since the latter can be presented through the contour integrals or the particular eigenvalue problems. Different approaches [20,21] are used; however, computational costs are high, which is essential in the derivation of fast BEM code.

Therefore, this paper utilizes previously developed novel boundary integral equations [22] for obtaining the dual BEM for TMEE solids containing 3D cracks. All kernels are obtained explicitly. The issues on the efficient numerical evaluation of kernel functions, integration of singular and hypersingular integrals and accurate determination of field intensity factors are discussed in details.

2. Governing equations of heat conduction and thermomagnetoelectroelasticity

According to [22], in a fixed Cartesian coordinate system $Ox_1x_2x_3$ the equilibrium equations, the Maxwell equations (Gauss theorem for electric and magnetic fields), and the balance equations of heat conduction in the steady-state case can be presented in the following compact form

$$\tilde{\sigma}_{Ij,j} + \tilde{f}_I = 0, \quad h_{i,i} - f_h = 0, \tag{1}$$

where the capital index varies from 1 to 5, while the lower case index varies from 1 to 3, i.e. $I = 1, 2, \dots, 5$; $i = 1, 2, 3$. Here and further the Einstein summation convention is used. A comma at subscript denotes differentiation with respect to a coordinate indexed after the comma, i.e. $u_{i,j} = \partial u_i/\partial x_j$.

In the assumption of small strains and fields' strengths the constitutive equations of linear thermomagnetoelectroelasticity in the compact notations are as follows [22]

$$\tilde{\sigma}_{Ij} = \tilde{C}_{IjKm}\tilde{u}_{K,m} - \tilde{\beta}_{Ij}\theta, \quad h_i = -k_{ij}\theta_{,j}, \tag{2}$$

where

$$\begin{aligned}
&\tilde{u}_i = u_i, \quad \tilde{u}_4 = \phi, \quad \tilde{u}_5 = \psi; \quad \tilde{f}_i = f_i, \quad \tilde{f}_4 = -q, \quad \tilde{f}_5 = b_m;\\
&\tilde{\sigma}_{ij} = \sigma_{ij}, \ \tilde{\sigma}_{4j} = D_j, \quad \tilde{\sigma}_{5j} = B_j;\\
&\tilde{C}_{ijkm} = C_{ijkm}, \ \tilde{C}_{ij4m} = e_{mij}, \ \tilde{C}_{4jkm} = e_{jkm}, \ \tilde{C}_{4j4m} = -\kappa_{jm},\\
&\tilde{C}_{ij5m} = h_{mij}, \ \tilde{C}_{5jkm} = h_{jkm}, \ \tilde{C}_{5j5m} = -\mu_{jm},\\
&\tilde{C}_{4j5m} = -\gamma_{jm}, \ \tilde{C}_{5j4m} = -\gamma_{jm};\\
&\tilde{\beta}_{ij} = \beta_{ij}, \ \tilde{\beta}_{4j} = -\chi_j, \ \tilde{\beta}_{5j} = \nu_j \quad (i, j, k, m = 1, 2, 3);
\end{aligned} \tag{3}$$

σ_{ij} is a stress tensor; f_i is a body force vector; D_i is an electric displacement vector; q is a free charge volume density; B_i is a magnetic induction vector; b_m is a body current; h_i is a heat flux; f_h is a distributed heat source density; u_i is a displacement vector; ϕ, ψ are the electric and magnetic potentials, respectively; θ is a temperature change with respect to the reference temperature; C_{ijkm} are the elastic stiffnesses (elastic moduli); k_{ij} are heat conduction coefficients; e_{ijk}, h_{ijk} are piezoelectric and piezomagnetic constants; κ_{ij}, μ_{ij}, γ_{ij} are dielectric permittivities, magnetic permeabilities and electromagnetic constants, respectively; β_{ij}, χ_i and ν_i are thermal moduli, pyroelectric coefficients and pyromagnetic coefficients, respectively.

According to [22], the extended magnetoelectroelastic tensor $\tilde{C}_{IjKm}$ has the following useful symmetry property

$$\tilde{C}_{IjKm} = \tilde{C}_{KmIj}. \tag{4}$$

Thus, the problem of linear thermomagnetoelectroelasticity is to solve partial differential equations (1) and (2) under the given boundary conditions and volume loading. Since magneto-electro-mechanical fields do not influence temperature field in the considered problem (uncoupled thermomagnetoelectroelasticity) the first step is to solve the heat conduction equation and the second one is to determine mechanical, electric and magnetic fields acting in the solid.

3. Hypersingular boundary integral equations for 3D thermomagnetoelectroelasticity

Recently, novel truly boundary integral formulae were obtained for 3D anisotropic heat conduction and thermomagnetoelectroelasticity [22]

$$\theta(\mathbf{y}) = \iint_{\partial\mathfrak{B}} (\Theta^*(\mathbf{x}, \mathbf{y})h_n(\mathbf{x}) - H^*(\mathbf{x}, \mathbf{y})\theta(\mathbf{x}))dS(\mathbf{x}) - \iiint_{\mathfrak{B}} \Theta^*(\mathbf{x}, \mathbf{y})f_h(\mathbf{x})dV(\mathbf{x}), \tag{5}$$

$$\begin{aligned}
\tilde{u}_I(\mathbf{y}) = &\iint_{\partial\mathfrak{B}} (U_{IJ}(\mathbf{x}, \mathbf{y})\tilde{t}_J(\mathbf{x}) - T_{IJ}(\mathbf{x}, \mathbf{y})\tilde{u}_J(\mathbf{x}))dS(\mathbf{x})\\
&+ \iint_{\partial\mathfrak{B}} [R_I(\mathbf{x}, \mathbf{y})\theta(\mathbf{x}) + V_I(\mathbf{x}, \mathbf{y})h_n(\mathbf{x})]dS(\mathbf{x})\\
&+ \iiint_{\mathfrak{B}} U_{IJ}(\mathbf{x}, \mathbf{y})\tilde{f}_J(\mathbf{x})dV(\mathbf{x}) - \iiint_{\mathfrak{B}} V_I(\mathbf{x}, \mathbf{y})f_h(\mathbf{x})dV(\mathbf{x}),
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\Theta^*(\mathbf{x}, \mathbf{y}) = -\frac{1}{8\pi^2|\mathbf{x} - \mathbf{y}|} \oint_{|\boldsymbol{\lambda}|=1} (k_{ij}\lambda_i\lambda_j)^{-1} dl(\boldsymbol{\lambda}),\\
&H^*(\mathbf{x}, \mathbf{y}) = -k_{ij}n_i(\mathbf{x})\Theta^*_{,j}(\mathbf{x}, \mathbf{y});
\end{aligned} \tag{7}$$

$$U_{IJ}(\mathbf{x}, \mathbf{y}) = \frac{1}{8\pi^2|\mathbf{x} - \mathbf{y}|} \oint_{|\boldsymbol{\lambda}|=1} \Gamma_{IJ}^{-1}(\boldsymbol{\lambda})dl(\boldsymbol{\lambda}), \quad T_{QI}(\mathbf{x}, \mathbf{y}) = \tilde{C}_{IjKm}n_j(\mathbf{x})U_{QK,m}(\mathbf{x}, \mathbf{y}); \tag{8}$$

$$V_I(\mathbf{x}, \mathbf{y}) = \frac{1}{8\pi^2} \iint_{\substack{|\boldsymbol{\xi}|=1,\\ \boldsymbol{\xi}\cdot\mathbf{t}>0}} \frac{\Gamma_{IJ}^{-1}(\boldsymbol{\xi})\tilde{\beta}_{Jk}\xi_k}{k_{qr}\xi_q\xi_r} dS(\boldsymbol{\xi}), \quad R_I(\mathbf{x}, \mathbf{y}) = k_{mj}V_{I,j}(\mathbf{x}, \mathbf{y})n_m(\mathbf{x}), \tag{9}$$

where $\partial\mathfrak{B}$ is a boundary of the domain $\mathfrak{B}$ occupied by the solid; n_p is a unit outwards normal vector to the surface $\partial\mathfrak{B}$; $\tilde{t}_I = \tilde{\sigma}_{Ij}n_j$ is an extended traction vector; $h_n = h_in_i$; $\mathbf{t}$ is a unit vector collinear with $\mathbf{x} - \mathbf{y}$; and $\boldsymbol{\lambda}$ is a unit vector normal to $\mathbf{x} - \mathbf{y}$; $\mathbf{y}$ is an internal point in the domain $\mathfrak{B}$; $\Gamma_{IJ}^{-1}(\boldsymbol{\xi})$ are the components of the matrix, which is inverse of the matrix $\Gamma_{IK}(\boldsymbol{\xi}) = \tilde{C}_{IjKm}\xi_j\xi_m$, i.e. $\Gamma_{IK}^{-1}(\boldsymbol{\xi})\Gamma_{KJ}(\boldsymbol{\xi}) = \delta_{IJ}$. Here and further the derivatives are evaluated for the variables x_i.

Integral formulae are the basis for derivation of the boundary integral equations, which replaces the boundary value problem for partial differential equations (1) and (2). Taking the limit when internal point $\mathbf{y}$ approaches the boundary point $\mathbf{x}_0 \in \partial\mathfrak{B}$ in the assumption that the boundary $\partial\mathfrak{B}$ is smooth at $\mathbf{x}_0$ one obtains

$$\begin{aligned}
\tfrac{1}{2}\theta(\mathbf{x}_0) = &\iint_{\partial\mathfrak{B}} \Theta^*(\mathbf{x}, \mathbf{x}_0)h_n(\mathbf{x})dS(\mathbf{x}) - \mathrm{CPV}\iint_{\partial\mathfrak{B}} H^*(\mathbf{x}, \mathbf{x}_0)\theta(\mathbf{x})dS(\mathbf{x})\\
&- \iiint_{\mathfrak{B}} \Theta^*(\mathbf{x}, \mathbf{x}_0)f_h(\mathbf{x})dV(\mathbf{x}),
\end{aligned} \tag{10}$$

$$\begin{aligned}
\tfrac{1}{2}\tilde{u}_I(\mathbf{x}_0) = &\iint_{\partial\mathfrak{B}} U_{IJ}(\mathbf{x}, \mathbf{x}_0)\tilde{t}_J(\mathbf{x})dS(\mathbf{x}) - \mathrm{CPV}\iint_{\partial\mathfrak{B}} T_{IJ}(\mathbf{x}, \mathbf{x}_0)\tilde{u}_J(\mathbf{x})dS(\mathbf{x})\\
&+ \iint_{\partial\mathfrak{B}} [R_I(\mathbf{x}, \mathbf{x}_0)\theta(\mathbf{x}) + V_I(\mathbf{x}, \mathbf{x}_0)h_n(\mathbf{x})]dS(\mathbf{x})\\
&+ \iiint_{\mathfrak{B}} U_{IJ}(\mathbf{x}, \mathbf{x}_0)\tilde{f}_J(\mathbf{x})dV(\mathbf{x}) - \iiint_{\mathfrak{B}} V_I(\mathbf{x}, \mathbf{x}_0)f_h(\mathbf{x})dV(\mathbf{x}),
\end{aligned} \tag{11}$$

where CPV stands for the Cauchy Principal Value of the integral. These equations allow obtaining the unknown boundary functions, which are not set by the boundary conditions. Thereafter, when all boundary functions are known Eqs. (5) and (6) allow determining thermal, magnetic, electric and mechanical fields at an arbitrary internal point of the solid.

Nevertheless, integral equations (10) and (11) degenerate, when the boundary $\partial\mathfrak{B}$ or its part has the shape of a mathematical cut [23]. In this case both displacement and traction boundary integral equations should be used. Therefore, one should utilize Eqs. (2), (5) and (6) to derive heat flux and extended stress integral formulae and then apply limiting procedure to obtain heat flux and extended traction integral