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Boundary element analysis of bar silencers using the scattering matrix with two-dimensional finite element modes

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ABSTRACT

Bar silencers used in industry may consist of a large array of rectangular or round bars packed in a rectangular lattice arrangement. Due to the size of the lattice, normally only a single unit that represents a building block for the lattice is isolated for analysis purposes. Even with one isolated unit, the inlet and the outlet are still quite large, and the plane-wave cutoff frequency can be very low. Therefore, higher-order modes must be considered at the inlet and outlet in order to calculate the transmission loss. This paper uses the recently developed “impedance-to-scattering matrix method” to convert the element-based impedance matrix into the mode-based scattering matrix for transmission loss calculation. Depending on the shape of the inlet and outlet, it may not always be possible to find an analytical expression of the modes needed for the modal expansion. In this paper, the two-dimensional finite element method is used to extract the eigenvalues and the eigenvectors of the inlet/outlet cross section. The eigenvectors are then used in the modal expansion to convert the impedance matrix into the scattering matrix. Test cases include several commonly used inlet and outlet configurations, such as rectangular, circular and triangular cross sections.

1. Introduction

Dissipative silencers are widely used to attenuate broadband noise generated from gas turbine engines, power plants and HVAC ducts. There are a variety of designs of dissipative silencers distributing sound absorbent in various ways. A very simple design called the lined duct design places the sound absorbing material around the circumference of the cross-sectional area. Another prevailing design is the splitter silencer or the parallel-baffle silencer, which arranges multiple sound absorbing baffles parallel to the direction of exhaust. Normally, the sound attenuation of such silencers is proportional to the perimeter-area ratio and length [1]. There have been plenty of research activities on lined-duct silencers or splitter silencers either analytically, numerically or experimentally [2–7]. In 1983, Nilsson and Söderqvist proposed the idea of bar silencers and claimed that an array of square bars made of sound absorbing materials have certain advantages over a similarly configured splitter silencer [8]. In 1996, Cummings and Astley [9] investigated the acoustical behavior of square bar silencers using the finite element method (FEM) and compared to the experimental data.

To date, the FEM is still the most developed and widely used numerical technique for large dissipative silencers. Kirby and his co-workers [10–14] used a hybrid analytical-FEM to study the acoustical performance of various large dissipative silencers. To apply the hybrid technique, the 2D FEM is first employed to extract the eigenvalues and the associated eigenvectors of an axially uniform cross section. These 2D transversal modes are then used in the modal expansion along the axial direction if the cross section remains the same. To determine the unknown amplitudes in the modal expansion, either a point collocation method or an integral-based mode matching scheme is adopted to enforce the continuity of sound pressure and particle velocity at both ends where the uniform section meets the flanges or any irregular junctions. Since the FEM is mainly used on a 2D cross section to extract the modes, the hybrid analytical-FEM is a very efficient numerical technique for silencers with a very long axially uniform section.

As a viable alternative to the FEM, the boundary element method (BEM) has also been used for muffler and silencer analysis [15–17]. The direct mixed-body BEM in Refs. [15–17] can handle complex internal components, such as thin bodies, perforated tubes, and multiple bulk-reacting materials, in one single BEM domain without resorting to the multi-domain approach. It is well known that the BEM has the advantage of modeling the surface only. However, for a large silencer, even the surface mesh may still contain too many elements for

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a desktop computer to run comfortably. Normally, one way to solve large-scale problems in BEM is to use the fast multipole method (FMM) [18,19]. The FMM is especially useful for exterior radiation and scattering problems. However, the FMM has not been extended to silencer problems with complex internal components and multiple bulk-reacting materials yet. Fortunately, the interior domain of a large silencer can always be divided into several smaller substructures by using the substructure BEM [20] so that each substructure can fit within the memory limitation of a desktop computer. Each substructure produces an impedance matrix, and a so-called "impedance matrix synthesis" procedure [20] is used to combine all the substructures.

Until very recently, the BEM analysis of mufflers and silencers has been limited to problems with a small inlet and a small outlet only due to its dependency on either the four-pole matrix or the anechoic termination boundary condition for transmission loss (TL) calculation [15,16]. Without a proper introduction of modal expansion into the BEM, it is difficult to model large silencers with many higher-order modes emerging at the inlet and outlet cross sections. Zhou et al. [21] recently proposed a reciprocal identity method in conjunction with the BEM impedance matrix to extract the higher-order modes at the inlet and outlet. Each reciprocal identity couples the analytical modal expansion in the inlet and outlet ducts to a BEM solution with a random boundary condition set. The BEM impedance matrix can naturally provide more than enough such solutions for the reciprocal identity coupling. The method can be regarded as an indirect post-processing filter applied to the BEM impedance matrix to extract the higher-order modes for TL calculation.

Parallel to the reciprocal identity method, a different technique called the "impedance-to-scattering matrix method" was recently developed by Wang and Wu [22] as a direct collocation approach to convert the element-based impedance matrix into the mode-based scattering matrix for TL computation. The BEM impedance matrix relates sound pressures at the inlet and outlet to the corresponding particle velocities, while the scattering matrix relates the modes at the inlet and outlet [23]. It should be noted that the term "scattering matrix" can be a little confusing and it is not the same as the T-matrix method for exterior scattering problems. The scattering matrix is a special tool designed for muffler and silencer analysis, and is equivalent to the four-pole matrix if the plane-wave assumption is still valid at the inlet and outlet. In the impedance-to-scattering matrix method [22], each sound pressure and particle velocity can be directly expanded in terms of the duct modes at the centroid of each constant boundary element. These point-wise expansions are then related by the BEM impedance matrix, and the scattering matrix can be obtained after a few matrix operations. Like the reciprocal identity method, the impedance-to-scattering matrix method can be regarded as a post-processing filter applied to the BEM impedance matrix. The main difference between these two methods is that the reciprocal identity method is an integral-based method, while the impedance-to-scattering matrix method is a direct collocation method. Both methods rely on an analytical modal expansion at the inlet and outlet. Normally this is not an issue for a rectangular or circular inlet/outlet. However, more complicated inlet/outlet configurations will require a numerical solution of the duct modes first.

In this paper, the impedance-to scattering matrix method by Wang and Wu [22] is expanded to include the use of numerical modes from the 2D FEM so that any irregular inlet/outlet configurations can be modeled. The function of the 2D FEM is to extract the eigenvalues and the associated eigenvectors (modes) of the inlet/outlet cross sections. The 2D FEM procedure is the same as the one used in the hybrid analytical-FEM [10–14] or the hybrid analytical-BEM [24], except that it is only done at the inlet/outlet cross sections as a post-processing tool to extract the duct modes. The proposed method is by no means a hybrid technique because the silencer structure itself is still modeled by the same substructure BEM as in Ref. [20] with the impedance matrix being the primary output from the BEM. Modal expansion and the associated 2D FEM analysis are only introduced in the end at the inlet and outlet cross sections after the resultant BEM impedance matrix is produced. Fig. 1 shows the flowchart of the proposed computation procedure.

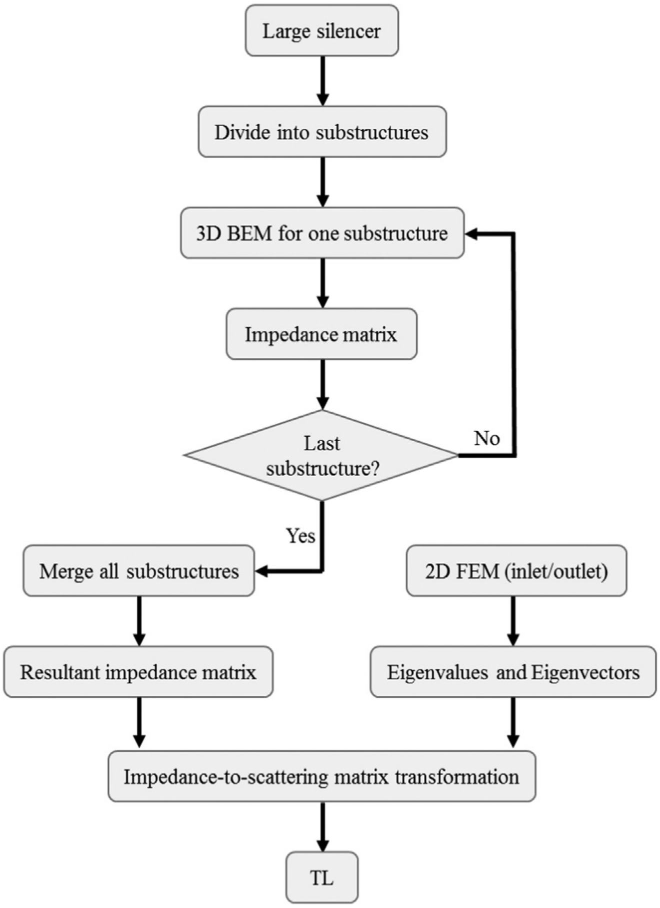

Fig. 1. Flowchart of the computation procedure.

Figs. 2–4 show three typical lattice arrangements of bar silencers. Usually a small building block is isolated from the lattice for analysis purposes. The building block in Figs. 2 and 3 is simply a round or rectangular bar housed in a rectangular duct. The duct walls are assumed rigid due to symmetry. The building block in Fig. 4 actually begins with a hexagon duct. Due to rotational symmetry, the hexagon eventually reduces to just a triangle. Although rectangular and circular

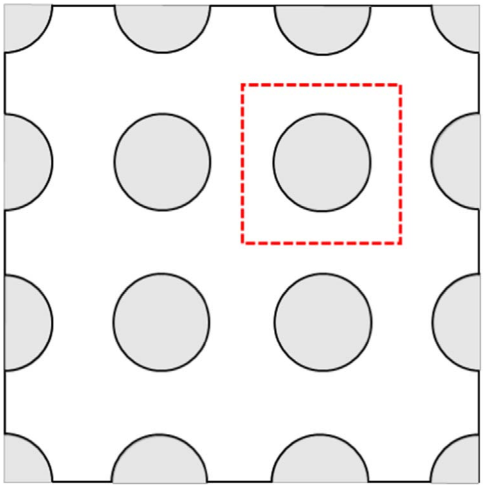

Fig. 2. A rectangular module isolated from an aligned lattice arrangement of round bars.