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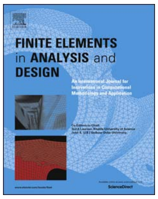

Three-dimensional constitutive modelling of arbitrarily orientated timber based on continuum damage mechanics

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ABSTRACT

An orthotropic constitutive model is developed in order to capture the complex response of unidirectional fibrous materials with arbitrary orientation under a three-dimensional (3D) stress state. Different modes of failure and their associated criteria are derived via decomposition of a single-surface yield function in the fibre and matrix directions. The uniaxial behaviour of each stress component is approximated by a multi-linear model, and the concepts of continuum damage mechanics are utilised to model the degradation of various components under a general 3D stress state. The proposed model is implemented as a user material subroutine in ABAQUS®, and it is utilised to analyse the non-linear mechanical response of bulk or laminated timber. The crack band model and the viscous regularisation techniques are employed to alleviate the spurious mesh sensitivity and convergence problems in the softening state. Some benchmark and practical examples are presented to verify the accuracy and performance of the proposed model for predicting different modes of failure and non-linear behaviour with respect to orientation of the fibres and loading.

1. Introduction

In the past decade, the introduction of new processing technologies has revolutionised timber construction and has introduced innovative applications using engineered wood. This resurgence of timber construction can be traced to the favourable intrinsic characteristics of wood as a natural, lightweight, and sustainable construction material that sequestrates CO_2 in the carbon cycle. However, a safe and reliable application of timber in demanding structural systems requires a thorough understanding of its complex behaviour, which is a result of its fibrous nature and its being directionally dependent. Therefore, the development of a comprehensive material model that can fully encapsulate the complex behaviour of timber and its expected modes of failure is much needed for both advanced research and industrial needs.

Timber is generally an orthotropic fibrous material that exhibits different mechanical properties in parallel-to and perpendicular-to the grain directions [1–5]. Its behaviour in tension can be described by linear elasticity followed by a brittle softening, regardless of the grain orientation [1–5]. Compression in the parallel-to-grain direction produces non-linear behaviour that is characterised by plasticity with a slight hardening and limited ductility up to the ultimate stress, followed by a mild softening region [5–7]; whilst perpendicular-to-grain compression may lead to wood densification that triggers sizeable hardening [4–8]. A realistic constitutive model must properly capture these failure modes and the resultant non-linear response under a multi-axial stress state for different fibre orientations. Existing constitutive laws for timber can be classified as empirical models [9], elasticity-based models [10,11], plasticity-based models especially for ductile failures [4,8,12–16], models for brittle failure based on continuum damage mechanics [16–19], fracture mechanics [20–23] or discrete element/lattice models [24,25]. Combined plasticity-damage or plasticity-fracture models are also of great interest, especially for representing the behaviour under cyclic loading.

These phenomenological representations are mainly built upon a representative failure criterion that describes the extent of the failure and the yield surfaces. The most-widely used failure criteria for orthotropic materials, such as those in [26–28], are single-surface yield functions that were developed mainly for ductile failure in metal alloys. These failure criteria have been utilised for non-linear modelling of timber structures such as in [12–15], but they are not able to predict effectively the different failure modes in fibrous materials [1–3]. In order to circumvent this deficiency, several approaches have been proposed, from which the definition of distinctive failure surfaces per quadrant of stress [1–3] and decomposition of a single-surface criterion in the fibre and matrix directions [29–32] are amongst the most favoured techniques. Multi-surface plasticity approaches have been applied successfully to wood structures [1–3]. Despite being more

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accurate than single-surface criteria, difficulties in implementation accompanied by sophisticated constitutive laws and the possibility of numerical instabilities have limited their widespread application [16,18,19]. A decomposition technique, such as the seminal model of Hashin [29] for unidirectional fibrous composites, is relatively more straightforward and physically meaningful, and produces continuous convex surfaces for each failure mode [16,18,19].

Applications of continuum damage mechanics (CDM) in modelling fibrous composites are widespread and well developed [33–39]. CDM is a simple and straightforward approach to model softening or even hardening behaviour *via* modification of the corresponding stiffness matrix, but it suffers from mesh dependency and convergence problems when dealing with the softening state [35,37]. To fully or partially restore this drawback, various methods have been introduced such as the crack band model [40], viscous regularisation technique [41] and non-local damage averaging models [42]. Some researchers have employed the concepts of CDM to model the mechanical behaviour of wooden structures [16–19]. Cofer et al. [17] utilised an anisotropic damage model which was based on some simplified assumptions to preliminarily investigate the potentials of this approach. Building on a plasticity-based model for compression and a continuum damage model for shear and tension, a three-dimensional (3D) material model for wood was developed for LS-DYNA finite element software [16]. Later, Sandhaas et al. [18,19] developed a 3D constitutive model by utilising continuum damage mechanics and implemented the proposed material model in ABAQUS.

In this study, an orthotropic 3D constitutive model is developed for fibrous materials and it is utilised for the non-linear analysis of timber members with arbitrary orientation of the grain direction. Moreover, different modes of failure and their representative failure criteria are obtained *via* a decomposition technique. The concepts of CDM are then utilised to model the stiffness degradation in various stress components and to evaluate the non-linear response under a multi-directional stress state. The proposed constitutive and damage models encompass some of the existing represent for brittle and ductile modes of failure as special cases. The proposed model is then implemented as a user material subroutine (UMAT) in ABAQUS software. Section 2 presents the material model and is followed by the derivation of a compatible continuum damage model in Section 3. The numerical analysis adopted with a supporting algorithm is described in Section 4. The accuracy and applicability of the material model is verified in Section 5, with a presentation of some benchmark or practical numerical examples. Section 6 concludes the paper with a brief summary of its accomplishments.

2. Material model

In this section, a material model is developed to express the non-linear behaviour of different stress components in a 3D timber element to predict the overall inelastic response of timber elements. Different modes of failure are expressed by series of representative failure criteria that are derived from decomposition of an orthotropic single-surface criterion in different material directions.

2.1. Non-linear behaviour

Timber is generally considered as an orthotropic material that shows directional dependency with its material properties differing in various orthogonal directions. A timber element with its 3D stress components and the main material directions, *i.e.* parallel-to-grain (longitudinal) and perpendicular-to-grain (radial and tangential) are shown in Fig. 1. Assuming that the material properties in the radial and tangential directions are identical or sufficiently close, timber may be treated as a transversely isotropic material.

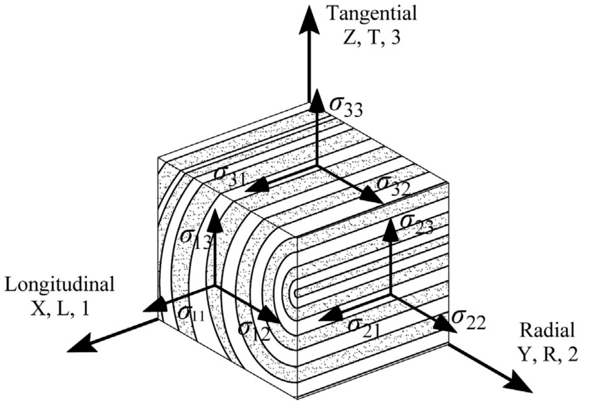

Fig. 1. 3D stress components and orthogonal material directions for a timber element.

The elastic behaviour of any material is expressed in tensor format by the equations

$$\varepsilon_i = C_{ij}\,\sigma_j \quad \forall\ i, j \in \{1, 2, \cdots, 6\}, \tag{1}$$

where ε_i and σ_j are the components of the engineering strain and stress vectors respectively, *i.e.* $\boldsymbol{\varepsilon} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{23}, 2\varepsilon_{12}, 2\varepsilon_{13}\}^{\mathrm{T}}$ and $\boldsymbol{\sigma} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{12}, \sigma_{13}\}^{\mathrm{T}}$. In addition, C_{ij} are the entries of the elastic compliance matrix ($\mathbf{C}$) that has the following form for an orthotropic material:

$$\mathbf{C} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_1} & -\frac{\nu_{31}}{E_1} & & & \\ -\frac{\nu_{12}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_2} & & 0 & \\ -\frac{\nu_{13}}{E_3} & -\frac{\nu_{23}}{E_3} & \frac{1}{E_3} & & & \\ & & & \frac{1}{E_4} & & \\ & 0 & & & \frac{1}{E_5} & \\ & & & & & \frac{1}{E_6} \end{bmatrix}, \tag{2}$$

where E_i are the entries of vector $\mathbf{E} = \{E_{11}, E_{22}, E_{33}, G_{23}, G_{12}, G_{13}\}^{\mathrm{T}}$ that represent the elasticity moduli for the different actions. Moreover, ν_{ij} are Poisson's ratios that satisfy $\nu_{12}E_1 = \nu_{21}E_2$, $\nu_{13}E_1 = \nu_{31}E_3$ and $\nu_{23}E_2 = \nu_{32}E_3$ to ensure the expected symmetry in elastic compliance matrix. The elastic stiffness matrix ($\mathbf{D}$) is then obtained as the inverse of elastic compliance matrix, *i.e.* $\mathbf{D} = \mathbf{C}^{-1}$.

Timber elements tend to exhibit complex responses due to strong directional dependency and material non-linearity when their elastic limits ere exceeded. Timber in compression typically is typified by plastic behaviour with hardening potential, whereas the tensile/shear response generally displays a brittle softening behaviour. Therefore, for each stress component, a general multi-linear model (Fig. 2) is assumed in this study to express the uniaxial stress-strain relationship of the timber element.

The material model compromises of an initial elastic state followed by a linear hardening and a subsequent bi-linear softening that are adjusted for different actions. This general behaviour can be simplified to some commonly-used models such as an elastoplastic model (see

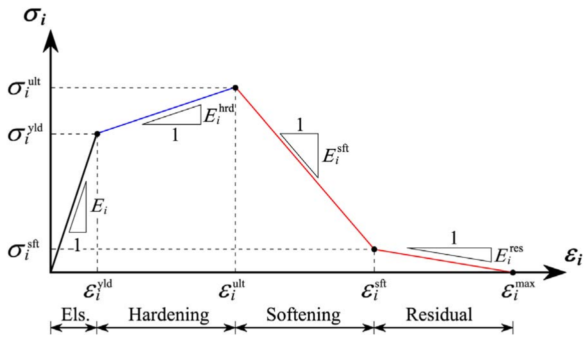

Fig. 2. Multi-linear material model assumed for each stress component.