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A multiscale fixed stress split iterative scheme for coupled flow and poromechanics in deep subsurface reservoirs

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ABSTRACT

In coupled flow and poromechanics phenomena representing hydrocarbon production or CO_2 sequestration in deep subsurface reservoirs, the spatial domain in which fluid flow occurs is usually much smaller than the spatial domain over which significant deformation occurs. The typical approach is to either impose an overburden pressure directly on the reservoir thus treating it as a coupled problem domain or to model flow on a huge domain with zero permeability cells to mimic the no flow boundary condition on the interface of the reservoir and the surrounding rock. The former approach precludes a study of land subsidence or uplift and further does not mimic the true effect of the overburden on stress sensitive reservoirs whereas the latter approach has huge computational costs. In order to address these challenges, we augment the fixed-stress split iterative scheme with upscaling and downscaling operators to enable modeling flow and mechanics on overlapping nonmatching hexahedral grids. Flow is solved on a finer mesh using a multipoint flux mixed finite element method and mechanics is solved on a coarse mesh using a conforming Galerkin method. The multiscale operators are constructed using a procedure that involves singular value decompositions, a surface intersections algorithm and Delaunay triangulations. We numerically demonstrate the convergence of the augmented scheme using the classical Mandel's problem solution.

1. Introduction

Solution schemes for coupled deformation–diffusion phenomena in porous media can be broadly classified into fully coupled, loosely coupled and iteratively coupled schemes. In a fully coupled scheme, the flow and mechanics equations are solved simultaneously at each time step [26,20]. The fully coupled approach is unconditionally stable, but requires careful implementation with substantial local memory requirements and specialized linear solvers. In a loosely coupled scheme, the coupling between flow and mechanics is resolved only after a certain number of flow time steps [25]. Such a scheme is only conditionally stable and requires a priori knowledge of the desired frequency of geomechanical updates leading to an accurate solution.

Iteratively coupled schemes are those in which an operator splitting strategy (see Armero and Simo [2], Schrefler et al. [27]) is used to split the coupled problem into flow and mechanics subproblems. At each time step, either the flow or mechanics problem is solved first, then the other problem is solved using the previous iterate solution information alternatively [30,23,24]. This sequential procedure is iterated until the solution converges to an acceptable tolerance. Carefully

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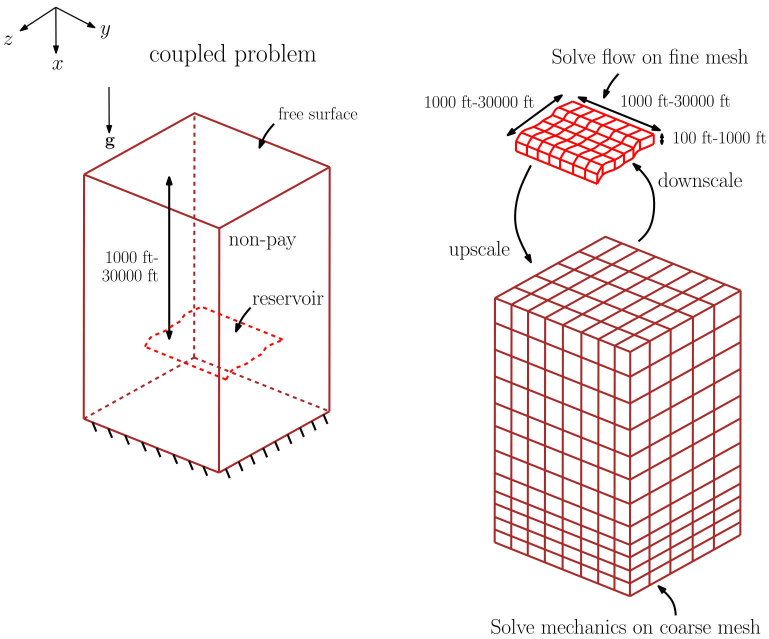

Fig. 1. Our multi-scale approach allows us to spatially decouple the flow and mechanics domains with different finite element discretizations. Typical dimensions are provided for the sake of clarity. A substantial underburden is also modeled to allow the reservoir to deform more naturally.

crafted convergence criteria lend solutions as accurate as that obtained using a fully coupled approach. Iteratively coupled algorithms have inherent advantages compared to fully coupled schemes from the standpoint of customization, software reuse and code modularity (see Felippa et al. [12]). A numerical comparison of the three techniques can be found in Dean et al. [11]. Kim et al. [21] studied the properties of a few operator splitting strategies for the coupled flow and poromechanics problem and recommended the fixed-stress split strategy where the flow problem is solved first while freezing the total mean stress. Later, Mikelić and Wheeler [23] rigorously proved the convergence of the fixed-stress split scheme using the principle of contraction mapping with appropriately chosen metrics. Castelletto et al. [7] showed that the fixed stress split strategy with a mixed formulation for flow coupled with a continuous Galerkin formulation for poromechanics is first order accurate in space and time for both pressure and displacement, even for distorted meshes. The reader is referred to White et al. [31] and Castelletto et al. [8] for a more general description of the link between a fully coupled scheme and iteratively coupled schemes in the context of preconditioning strategies to fully coupled schemes.

Previous attempts at solving the multi-scale problem include the works of Dean et al. [11], Gai et al. [15], Ita and Malekzadeh [19], Florez et al. [13] and Castelletto et al. [9]. Gai et al. [15] reformulated an iterative sequential scheme as a special case of a fully coupled approach and implemented the algorithm on overlapping nonmatching rectilinear grids but avoided 3D intersection calculations instead evaluating the displacement–pressure coupling submatrices using a mid-point integration rule. Florez et al. [13] implemented a procedure in which a saddle-point system with mortar spaces on nonmatching interfaces of a decomposed geomechanics domain is solved by applying a balancing Neumann–Neumann pre-conditioner. It involved subdomain to mortar and mortar to subdomain projections, Lagrange multiplier solve and parallel subdomain solves at each time step with computationally expensive subdomain solves. Castelletto et al. [9] implemented a procedure for nested grids in which coarse scale basis functions for the poromechanical solve are obtained in terms of fine scale basis functions by solving local equilibrium problems on each coarse scale poromechanical element. These coarse scale basis functions are then used to construct prolongation and restriction operators, which are then employed to construct a two-stage preconditioner for the coarse scale poromechanical solve.

As shown in Fig. 1, our multi-scale approach allows us to spatially decouple the flow and mechanics domains and model the flow subproblem on a fine grid and the mechanics subproblem on a coarse grid with a larger spatial extent. With regards to the flow problem restricted to the reservoir, we impose the no flow boundary condition obtained from first principles by Mikelić and Wheeler [22]. In this work, we augment the operator splitting scheme of Mikelić and Wheeler [23] with multiscale operators as shown in Fig. 2 and further demonstrate the numerical convergence of the augmented scheme using the Mandel's problem solution as a benchmark. The constructions of the multiscale operators are performed only once during the pre-processing step thus avoiding the expense of the mortar based method of Florez et al. [13]. To the best of our knowledge, this is the first time the concepts of discrete geometry are being used to solve the coupled flow and poromechanics problem on nonmatching distorted hexahedral grids. The paper is structured as follows: Section 2 presents the finite element formulation for flow and mechanics. Section 3 presents the augmented solution algorithm. Section 4 presents the details about the singular value decompositions, the surface intersections algorithm and Delaunay triangulations used in constructing the operators. In Section 5, we numerically demonstrate the convergence of the augmented scheme