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A numerical study on parasitic capillary waves using unsteady conformal mapping

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ABSTRACT

This paper describes fully nonlinear computation of unsteady motion of parasitic capillary waves that appear on the front face of steep gravity waves progressing on water of infinite depth, within the framework of irrotational plane flow. As an alternative to the widely-used boundary integral method with mixed-Eulerian–Lagrangian (MEL) time updating, we focus on a numerical method based on unsteady conformal mapping, which will be hereafter referred to as the unsteady hodograph transformation (UHT) method. In this method, we solve the nonlinear evolution equations to find an unsteady conformal map in a complex plane with which the flow domain is mapped onto the unit disk while the free surface is fixed on the unit circle. The aim of this work is to compare the UHT method with the MEL method and find a more efficient method to compute parasitic capillary waves. From linear stability analysis, it is found that a critical difference between these two methods arises from the kernel of cotangent function in singular integrals, and the UHT method can avoid some numerical instability due to it. Numerical examples demonstrate that the UHT method is more suitable than the MEL method for not only parasitic capillary waves, but also capillary dominated waves. In particular, the UHT method requires no artificial techniques, such as filtering, to control numerical errors, in these examples. In addition, another major difference between the two methods is observed in terms of the clustering property of sample points on the free surface, depending on the restoring force of waves (gravity or surface tension).

1. Introduction

We consider numerical computation of “parasitic capillary waves”, which are a train of short capillary waves generated on the forward face of steep gravity waves progressing on water of infinite depth. These waves have been studied extensively as they play important roles in the generation of wind waves and wave breaking (see review by Perlin & Schultz [39]). In addition, the complete understanding of parasitic capillary waves is necessary for accurate estimation of sea state from remote sensing of the sea surface elevation.

Since Cox’s experiments [14], considerable progress has been made in the study of parasitic capillary waves. Longuet-Higgins [30,31] developed a linear steady model with viscous damping by assuming that the parasitic capillary waves can be considered as a perturbation due to surface tension on progressive pure gravity waves. His theory agrees qualitatively with

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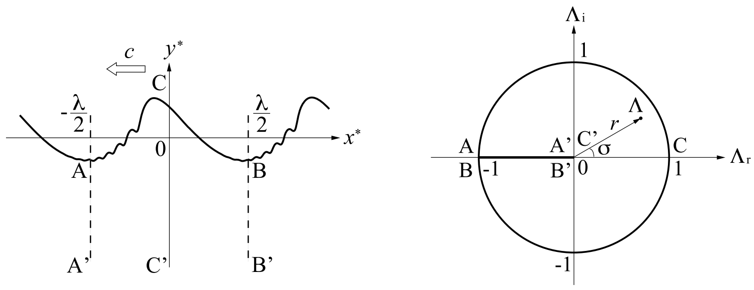

(a) The physical plane $(z = x + \mathrm{i}y)$

(b) The Λ-plane $(\Lambda = \Lambda_r + \mathrm{i}\Lambda_i = r e^{\mathrm{i}\sigma})$

Fig. 1. The two-dimensional motion of periodic water waves in the frame of reference moving with waves and conformal mapping of the flow domain onto the unit disk in the Λ-plane. The coordinates x^* and y^* are non-dimensionalized as $x = (2\pi/\lambda)x^*$ and $y = (2\pi/\lambda)y^*$ (see eq. (1)). c and λ are characteristic speed and length of waves, respectively.

some experiments [14,22,38]. Some additional effects such as wind forcing [23,24] or vortex motion [34,27] have also been considered. However, as Jiang et al. [28] remarked, the essential mechanism of generation of parasitic capillary waves is the nonlinear interaction between short capillary waves and long gravity waves in an inviscid flow. Also, while the steadiness assumption has been often adopted [30,31,23,24], the flow unsteadiness is non-negligible [19,28].

In this work, to catch this nonlinear and unsteady process, we focus on fully nonlinear computation of unsteady motion of gravity-capillary waves within the framework of irrotational plane flow in the vertical cross-section along the propagation direction of waves, as shown in Fig. 1(a). Here it should be noted that our main interest is in the generation and growth of capillary waves on primary gravity waves, which is essentially different from finding steady and symmetric solutions of gravity-capillary waves considered by Schwartz & Vanden-Broeck [42] and Chen & Saffman [10,11].

As a standard method of fully nonlinear computation of unsteady water waves, a boundary integral method with mixed-Eulerian–Lagrangian time updating has been well developed [50,1,18,4,40,26,3] (see reviews by Hou [25], Tsai & Yue [48], Dias & Bridges [17] and Perlin, Choi & Tian [37]) after Longuet-Higgins & Cokelet [32] proposed the method for two-dimensional breaking waves. This method is based on a boundary integral formulation for fully nonlinear potential flow problems with time updating using the free surface conditions in a mixed-Eulerian–Lagrangian form (eqs. (7) and (8) in section 2.1). We call this method the mixed-Eulerian–Lagrangian (MEL) method or just “MEL”. Validity of the MEL method has been examined experimentally [20,44] and theoretically [4,5,8]. Jiang et al. [28] applied this method to parasitic capillary waves, and found that nonlinearity and unsteadiness characterize this phenomenon. However, Beale et al. [5] and Hou [25] pointed out that the numerical simulations using this method are sensitive to numerical instabilities due to discretization of a singular integral and aliasing errors, as will be also discussed in sections 4.1 and 5.1 of this paper. Although a number of variations exist for MEL, we choose for our computation the approach of Beale et al. [4,5,25], whose numerical properties have been extensively studied.

Recently an alternative approach via time-dependent conformal mapping has been used to fully nonlinear computation of the two-dimensional motion of water waves [33,9,13,21,29,12,47] (see the review by Perlin, Choi & Tian [37, section 6.1]). In this method, the flow domain is conformally mapped onto a fixed domain such as a strip of uniform thickness or a unit disk in a complex plane, and the free surface is onto its boundary that remains unchanged in time, as shown in Fig. 1(b). The time-dependent conformal mapping functions and, therefore, the time evolution of the free surface in the physical plane can be determined by solving the surface Euler equations given later by eqs. (9) and (10) in section 2.2. We call this method the Unsteady Hodograph Transformation (UHT) method or just “UHT” as the formulation in the steady limit can be reduced to the classical hodograph transformation commonly used for steady waves, where the fluid domain is mapped into a uniform strip or a unit disk by interchanging the roles of dependent and independent variables.

It should be noticed that Longuet-Higgins & Cokelet [32] and Schultz, Huh & Griffin [40] also combined a conformal mapping technique with MEL to study breaking gravity waves. In their method, the still water level is mapped onto the unit circle using a stationary or time-independent mapping function, and the free surface moves in time in the mapped complex plane. On the other hand, in UHT, the whole flow domain is conformally mapped onto the unit disk using a non-stationary mapping function so that the free surface always stays on the unit circle in the mapped plane. Thus, evaluating a singular integral written in the mapped plane in UHT is much simpler than that in the physical plane in MEL, as shown in section 5.2. Also note that Tanveer [45,46] discussed time variation of singularities of a solution by conformally mapping the flow domain onto the unit disk and using analytic continuation of the free surface conditions.

In both methods, MEL and UHT, each dependent variable and its spatial derivative are spectrally estimated under periodic boundary conditions via a pseudo-spectral method based on Fast Fourier transform (FFT). Then, due to spectral approximation using finite Fourier series, aliasing errors are unavoidable when nonlinear problems are solved, as shown in section 3.1. Nevertheless, the UHT method has some advantages in reducing these numerical errors, as demonstrated in section 6. Therefore, in comparison with the MEL method, the UHT method is shown more suitable for capillary dominated flows, as discussed in section 7. Earlier Chalikov and Sheinin [9] simulated overturning of gravity waves using the UHT method