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Distributed control of flexible demand using proportional allocation mechanism in a smart grid: Game theoretic interaction and price of anarchy[☆]

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ABSTRACT

Demand side management leveraging flexibility of electric loads is a promising approach to reduce peak demand and to maintain supply–demand balance in future power grids with large scale renewable penetration. Distributed control is an important tool used for managing the demand side. In this paper, we develop a distributed method for coordinated control of flexible loads. In an intra-day setting, based on the supply schedule of thermal generators and predicted supply of renewable generators, a central control authority sets up price to maximize social welfare. This can be done using *proportional allocation mechanism* assuming that the consumers are price takers. If the consumers are *price anticipators*, the corresponding situation is modeled using non-cooperative game theory and existence of Nash equilibrium is established. Selfish behavior of agents in such a game theoretic set-up can reduce the efficiency of the overall system as compared to centralized control. In this paper, we are able to bound the efficiency loss for our game. The lower bound on the price of anarchy (PoA) for our game is found to be 0.75. For a non-cooperative game in a smart grid with flexible loads having different operating constraints and having utility functions that are non-separable with respect to time, this result is a new contribution to the demand response literature.

1. Introduction

Greenhouse gas emissions leading to climate change and long-term sustainability are some of the major reasons motivating adoption of renewable energy sources such as wind and solar into the electric energy system. Large-scale integration of wind and solar electric energy poses significant technological challenges. These energy sources are inherently uncertain (power generation not known in advance), intermittent (large fluctuations and ramps) and non-dispatchable (unable to follow a command). The term *variability* is used to represent these three characteristics [1]. This variability is a significant hurdle in integrating renewables at a large scale [2]. Balancing supply and demand, a critical requirement in power systems operations, becomes more challenging because of this inherent variability.

Increase of peak power demand as a result of economic and population growth, particularly in developing countries, is also a serious concern for the electric grid. Conventionally, the total installed power system generation, transmission and distribution capacity is dictated by the need to cater to the peak demand. Thus, power system infrastructure costs increase as peak demand increases. But this peak demand occurs only for a small fraction of time in a year [3].

A possible solution to address the increase of variability and/or peak power consumption is to deploy *demand side management* (DSM) or *demand response* (DR) program where consumers adjust their demand taking advantage of flexibility of their electric loads. Demand response program is considered to be a promising solution to integrate intermittent and uncertain renewable energy in today's electrical systems [4–7]. Albadi et al. [8] distinguish two types of DR programs: incentive based programs (IBP) and price based programs (PBP). The latter aim to reduce the demand by offering a higher price during peak periods and lower price during off-peak periods. These rates include among others, Time of Use (TOU) rate, Critical Peak Pricing (CPP), and Real Time Pricing (RTP). In [9], a study of the response of residential consumers to dynamic pricing of electricity has been accomplished by surveying fifteen experiments of dynamic prices. The conclusions of this study are that time-based rates is the most favorable case and including

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enabling technologies induce a drop in peak demand that may reach 27%–44%. Our approach to demand side management is a price based program maximizing social welfare of a group of flexible consumers and guaranteeing supply–demand balance. We aim to design a transparent market based approach and analyze the effect of behavior of the consumers on its efficiency.

1.1. Literature survey

In the traditional power systems, the generation is adjusted to meet the uncontrollable demand. But a paradigm shift in power system operations is underway where consumers will be incentivized to manage their demand by leveraging the flexibility of their loads such as electric vehicles (EV), plug-in hybrid vehicles (PHEV), air conditioning, heat pumps, water heaters, pool pumps, washers, dryers, refrigerators, etc. The concepts of DSM or DR in power systems operations utilize the potential benefits that can accrue from exploiting this flexibility in electric loads. Different types of electric loads and their potential in contributing to electrical power demand management have been analyzed in [10–12]. Development of new advanced sensing, computation, communication, and controls technologies will be needed to realize this potential. Datacenters are also considered as valuable resources for DSM programs to balance power supply and demand due to their huge energy consumption and their flexibility for temporal and spacial load distribution [13,14]. Stackelberg games are proposed to analyze DR incentive schemes for either geo-distributed [13] and colocation datacenters [14]. Datacenters and their tenants, in the case of colocation datacenters, respond to a price signal originated by the utility of the datacenter operator.

Modern electrical power systems are large-scale systems with thousands of consumers and loads. For these systems, it is difficult to have a single decision maker, because different agents can have different objectives. In this setting, distributed control can be used as a major tool to solve problems where a control authority sends a control signal (*e.g.* price of electricity) and consumers decide their consumption schedules according to some private utility function [15,16]. A strategy for assigning quantities in a distributed price-based framework is the proportional allocation mechanism. In this mechanism, the control authority calculates a price for all the consumers in such a way that the assigned quantity to each agent is proportional to the monetary value that the agent is willing to pay. This mechanism was applied by Kelly [17,18] in a traffic routing problem for a communication network. It has also been used to formulate a distributed EV charging control problem in [19] and to develop a coordinated charging policy for EVs, inspired by lottery scheduling in [20]. However, the mechanism has not been examined in a much broader system level setting in the smart grid context.

Consumer behavior plays a critical role in the implementation of demand response programs in distributed mode. The success of DSM programs requires active consumer participation. However, not every consumer behaves always as a rational agent. The Enron scandal in 2001 showed that market participants can manipulate prices [21] and the assumption of price taking consumers might not always be true. In [22,23], the role of consumer participation in the smart grid is analyzed using prospect theory to refine existing game-theoretic techniques.

Selfish behavior of agents in a non cooperative game leads to inefficiency with respect to the solution that maximizes system welfare. The loss of efficiency in a Cournot competition game can approach 100% [24]. Thus, it is crucial to design distributed control systems in such a way that the efficiency loss due to selfish behavior can be reduced and bounded. The term *price of anarchy* (PoA) has been coined as an index to measure this loss of efficiency. Deriving PoA bounds has become a topic of great research interest in different fields such as economics, communication and computer science [25–28]. Bounds on the PoA for various cost sharing games, congestion games and pay-off maximization games have been derived in [25,26]. A tight PoA bound, *i.e.* Pigou bound for routing games, is derived in [27]. A tight bound on the PoA in the problem of allocating a single infinitely divisible resource among multiple competing users is obtained in [28]. Finally, robust bounds on the PoA for *smooth games* are reported in [26].

In the smart grid scenario, though non-cooperative game theoretic methods have been used to model problems [29–32], the loss of efficiency by selfish behavior has not been widely investigated. The non-cooperative behavior of agents in a dynamic oligopoly smart grid market structure has been shown to lead to suboptimal outcome in comparison to cooperative behavior [31], but no bound on the PoA has been derived. The Nash equilibrium has been shown to be efficient in an infinite population game when the charging rates of all the EVs are equal [29], however these assumptions are rather impractical. The Nash equilibrium game has been shown to be efficient in a demand response problem with different consumers, but the utility functions of the consumers were ignored [32]. Considering wind variability, a game has been formulated among various power consumers in [30] where the PoA bound has been calculated for an example case. Game theoretic modeling with a Nash equilibrium is only useful when the performance at Nash equilibrium is shown to be good with respect to centralized control. In a demand response model with Cournot competition game setting, we were able to achieve a robust lower bound of 50% on the PoA [33].

1.2. Our work with key technical contribution

In this paper, we propose a distributed method for controlling the flexible demand of the consumers with intra-day forecasts. Flexible consumers are modeled as individually rational agents that maximize their net utilities while satisfying their consumption constraints. The consumers bid the monetary value they are willing to pay at each time interval and a central control authority computes a price signal based on proportional allocation mechanism. We analyze two scenarios: *price taking* and *price anticipating* consumers. In the first case, we prove that if the control authority uses a proportional allocation method, a competitive equilibrium that maximizes the system welfare is achieved. Moreover, this competitive equilibrium is equivalent to the optimal solution obtained by dual Lagrangian decomposition of the centralized control problem. However, since the consumers are aware that they can improve their individual welfare by anticipating the effect of their (and others') decisions on the system price. In this second case, the selfish behavior of the consumers is modeled using a non-cooperative Cournot game and we prove that a Nash equilibrium exists. The Nash equilibrium is inefficient and we are able to obtain a lower bound on the PoA of 75%, i.e., the loss of efficiency is no greater than 25%. We also show that efficiency can be improved by either recruiting consumers with similar utility functions, classifying them into groups of similar utility, or increasing the number of flexible consumers. Some preliminary results were reported in our previous paper [34], where only electrical vehicles were considered as flexible loads with utility functions that are separable in time. In this paper, we propose a more general model of various loads using utility functions that are not separable in time. There are many existing demand response models where the consumers bid their power demand and price is set up either by using marginal cost or some function of total demand and/or supply [15,16,29,30]. Unlike those models, in our case consumers bid the monetary value and the control authority uses that bid to generate a system price by using proportional allocation. This allocation mechanism produces a highly efficient game as described in [28]. The results