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Assessment of optimal distribution network reconfiguration results using stochastic dominance concepts

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HIGHLIGHTS

- Comparing the results from heuristic optimization algorithms requires appropriate metrics.
- Rough metrics are used very often in the power and energy area.
- A more significant metric based on first-order stochastic dominance is proposed.
- The effectiveness of this metric is shown for distribution system optimal reconfiguration.

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ABSTRACT

In the recent research on power and energy system optimization, many deterministic and heuristic solvers have been proposed. Each proposal claims that the new solver is better than the previous ones on the basis of performance indicators, which are often limited to the best solution found or to simple statistics (mean, median, standard deviation). This paper introduces a new and more significant performance indicator based on the concept of first-order stochastic dominance. This indicator can generally compare the solutions of a given optimization problem for which the global optimum is not known. The optimal discrete distribution network reconfiguration for a real-scale system was taken as an example problem, to show the characteristics of the proposed indicator. The results obtained show the effectiveness of the proposed indicator to limit the acceptability of "better" solvers to the ones that actually exhibit enhanced performance with respect to incrementally improved benchmarks.

1. Introduction

In many optimization problems for power and energy system optimization (and to other fields as well), the number of possible solutions is so high that there is no way to ensure that the global optimum has been found. For some discrete optimization problems (e.g., distribution system optimal reconfiguration [1], or optimal planning [2]) only the number of possible solutions can be computed; however, not all the solutions can be generated in practice, as an exhaustive search is computationally intractable. For these problems, a number of pseudo-optimal solutions are obtained by running either different algorithms, or the same algorithm with different parameters. For algorithms whose outcomes depend on random number extractions, different solutions can also be found with the same parameters, just by changing the seed for the random number extraction at each execution. Recently, there has been a fast increase in the proposal of algorithms based on meta-heuristics to solve optimization problems. In many cases, they adopt rearranged versions of existing concepts without truly providing significant contributions or innovative ideas [3] and use simplistic performance indicators to state the superiority of the proposed algorithm over other algorithms.

Indeed, a basic question is *how* to compare the results coming from different algorithms. Various responses have been given in the literature. For example, a method can be claimed to be better than others if the best solution obtained with this method is better than all the solutions obtained from other methods. However, this is a rather limitative view. In fact, the global optimum could be found by chance during the random search. To increase the knowledge about the quality of the solver, simple statistical results (such as mean value, median, and standard deviation) have been adopted. However, these results lack specific information about the probabilistic distribution of the solutions.

In general, many publications in the power and energy systems area only provide a rough assessment of the statistical properties

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of the solutions obtained by the optimization algorithms. More significant assessments are carried out in the evolutionary computation domain [4] through non-parametric tests, which calculate the confidence interval around the mean (or the median) of the solutions [5]. A comparison of algorithms using Pareto-dominance analysis was introduced in [6], using two criteria (the number of function evaluations before stopping, and the objective function value), and applying an interval comparison scheme asymptotically corresponding to the first-order stochastic dominance condition.

The literature in the optimization community addresses the comparison among algorithms by running the algorithms on a number of *selected problems* and using *performance metrics* such as:

- The *performance profile* [7] of a solver, which is constructed as the Cumulative Distribution Function (CDF) of the performance ratio (i.e., the ratio between the computation time of the solver and the lowest computation time of all the solvers), and is then used to compare different solvers.
- The *data profile* [8], constructing the CDF of the problems that can be solved (within a specified target on the objective function value) with a given number of function evaluations, characterizes the performance of the solvers in computationally expensive optimization problems.

For both performance profiles and data profiles, higher values indicate a better performance. The computation time and the number of function evaluations are used as quantitative and interpretable results, expressing how far a solver behaves better than another. With these approaches, some challenges have to be overcome, such as the choice of the set of problems to analyse (no general criteria are defined), the setup of the target value (generally user-defined), and the handling of executions that fail to find the target value.

For a given optimization problem solved by using algorithms based on meta-heuristics, the nature of the solutions produced could be more significant than the computation time or the number of function evaluations to assess the performance of the algorithms, as good solutions could be obtained with very different computation times. In addition, when the relevant outcomes are the best solutions obtained from the algorithms, focusing the analysis on the confidence intervals around the mean or median could be less useful than providing more details on the occurrence of the best solutions.

On these bases, the contribution presented here aims to compare the performance of different optimization algorithms used to solve problems when the global optimum is not known a priori and may not even be found during the optimization process. The aim is to exploit the concept of *stochastic dominance* to construct a new performance indicator by considering the probabilistic distributions of the solutions found with different algorithms. This approach effectively highlights the relative importance of the best solutions obtained by the algorithms under test.

2. The stochastic dominance framework

Stochastic dominance concepts have been set up on the basis of the definitions provided by Hadar and Russell [9]. In this section, these concepts are adapted to the optimization problem outcomes. In particular, let us consider a problem with an arbitrary number of variables included in the vector $\boldsymbol{x}$, and an arbitrary number of parameters included in the vector $\boldsymbol{p}$. Without the loss of generality, the *minimization* of the single-objective function $g(\boldsymbol{x}, \boldsymbol{p})$ is considered with given sets of equality constraints $\boldsymbol{r}(\boldsymbol{x}, \boldsymbol{p}) = 0$ and inequality constraints $\boldsymbol{s}(\boldsymbol{x}, \boldsymbol{p}) \le 0$, as follows:

$$\begin{aligned} & y = \min\{g(\boldsymbol{x}, \boldsymbol{p})\} \\ \text{s.t.} \quad & \boldsymbol{r}(\boldsymbol{x}, \boldsymbol{p}) = 0 \\ & \boldsymbol{s}(\boldsymbol{x}, \boldsymbol{p}) \le 0. \end{aligned} \tag{1}$$

The optimization can be solved with different methods. Let us consider a generic number M of optimization methods applied to the same variables and parameters. For each method, the optimization (1) is solved for a given number H of executions, obtaining H outcomes of the variable y; these outcomes are sorted in the ascending order to construct the CDF of the solutions obtained. This CDF, obtained by running H executions of the method m, is denoted as $F_m^H(y)$.

Stochastic dominance is defined by considering a pair of optimization methods (e.g., m_1 and m_2), with H results obtained from each method, as follows:

1. *First-order* stochastic dominance (Fig. 1(a)): the method m_1 exhibits first-order stochastic dominance over the method m_2 if and only if $F_{m_1}^H(y) \ge F_{m_2}^H(y)$ for any y, with strict inequality holding at least for one value of y (to avoid the case with identical CDFs).
2. *Second-order* stochastic dominance (Fig. 1(c)): the method m_1 exhibits second-order stochastic dominance over the method m_2 if and only if the condition (2) holds for any y, with strict inequality existing at least for one value of y:

$$A_{m_1,m_2}^H(y) = \int_{z=0}^{y} \left(F_{m_1}^H(z) - F_{m_2}^H(z)\right) \ge 0. \tag{2}$$

The first-order stochastic dominance is a sufficient condition to guarantee second-order stochastic dominance, but not vice versa; in fact, the second-order stochastic dominance is based on an integral (not point-to-point) inequality and may also occur in some cases with intersections between the CDFs. With reference to Fig. 1, the solution from method m_2 is second-order stochastically dominated by both solutions of method m_1 and method m_3 (as the areas in Fig. 1(c) never become negative); however, the method m_3 does not exhibit first-order stochastic dominance over the method m_2 (as shown in Fig. 1(b)).

On these bases, the concept of first-order stochastic dominance is useful to develop a performance indicator that can be used for appropriately comparing the outcomes of two (or more) optimization methods. For this purpose, at first it is necessary to construct a *reference CDF* such that no other CDF can be found with one or more values located at the left of it (i.e., with better cumulative values, in the minimization case), as indicated in the following section.

3. Construction of the reference CDF

Let us consider a minimization problem solved by using M methods. Each method is run to obtain K solution points. The best H points, with $H \le K$, are then ordered in the ascending order to obtain the corresponding CDF, which is composed of H vertical steps of width $\Delta c = 1/H$ each.

Fig. 2(a) shows a qualitative example based on the results of two methods (denoted as m_{A} and m_{B}), with $H = 20$, already represented in CDF form. A temporary vector is constructed by merging all the solution points obtained from the different methods and by ordering them in the ascending order (Fig. 2(b)). The H solutions with the better (lowest, for minimization) values of the objective function are used to form the reference CDF $F_{\mathrm{ref}}^{(H)}$. Thereby, the reference CDF is composed of H vertical steps of width Δc each (Fig. 2(c)). The other points of the temporary vector are discarded. The choice of the two values H and K depends on the