Robust optimization of distance-based tolls in a network considering stochastic day to day dynamics

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ARTICLE INFO

ABSTRACT

This paper investigates the nonlinear distance-based congestion pricing in a network considering stochastic day-to-day dynamics. After an implementation/adjustment of a congestion pricing scheme, the network flows in a certain period of days are not on an equilibrium state, thus it is problematic to take the equilibrium-based indexes as the pricing objective. Therefore, the concept of robust optimization is taken for the congestion toll determination problem, which takes into account the network performance of each day. First, a minimax model which minimizes the maximum regret on each day is proposed. Taking as a constraint of the minimax model, a path-based day to day dynamics model under stochastic user equilibrium (SUE) constraints is discussed in this paper. It is difficult to solve this minimax model by exact algorithms because of the implicity of the flow map function. Hence, a two-phase artificial bee colony algorithm is developed to solve the proposed minimax regret model, of which the first phase solves the minimal expected total travel cost for each day and the second phase handles the minimax robust optimization problem. Finally, a numerical example is conducted to validate the proposed models and methods.

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1. Introduction

Congestion pricing, as an important instrument on transportation demand management, is of great significance in ameliorating urban traffic congestions in that it encourages commuters to adjust their travel behaviors: number of trips, route, time of day, destination, mode of transport, and so on, as well as the long-term decisions on where to live, work and set up business (de Palma and Lindsey, 2011). Among all types of congestion pricing schemes (zonal-based, cordon-based, distance-based, time-based as well as congestion-based schemes), the distance-based schemes have received increasing attention both academically and practically (e.g., Lawphongpanich and Yin, 2012; Daganzo and Lehe, 2015). Due to the better equity and efficiency of distance-based pricing, the current cordon-based congestion pricing scheme in Singapore will be upgraded to the distance-based pricing scheme, which is regarded as the next generation of Electronic Road Pricing (ERP) system from 2020 onwards (Singapore LTA, 2013). The optimal toll design problem is of considerable significance for improving the efficiency of the network. Generally, system wide indexes such as the total travel cost (TTC) are taken as the objective of the optimal toll design problem.

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Nearly all the existing studies use the equilibrium flow to calculate the TTC, and then evaluate each toll pattern based on the calculated TTC. However, any new toll pattern will affect travelers' route choice decisions, and the network flows cannot achieve an equilibrium state overnight. Cho and Hwang (2005) tested a small numerical network and revealed that it nearly takes 200 days to reach equilibrium state, thus it would take much longer time in a big urban area to achieve equilibrium. In addition, after such a long period, the network demand and infrastructure are largely changed, thus a new design of the optimal toll is needed again. Hence, in the whole study period of an optimal toll design problem (denoted by D), the day-to-day models can better capture the network flow conditions, rather than the final equilibrium state (He et al., 2010). Note in passing that in practice, to avoid the confusions from travelers on the toll, it is necessary to implement an unchanged toll in the whole period D; for instance, Singapore's ERP toll is adjusted every three months (Olszewski and Xie, 2005; Liu et al., 2013), and kept unchanged in-between, thus D equals three months in this case.

During the planning horizon D, the TTC is changing each day due to the change of traffic flows. Therefore, no toll pattern can give rise to a minimal TTC in each day of D. It is not reasonable to implement the toll pattern that gives rise to the minimal TTC on a certain day while neglecting the other days. From the viewpoints of policy-makers, the deterioration of some worst cases is more harmful than the loss of efficiency on the good cases, both temporally and spatially. The most desired toll pattern is the one that considers the traffic conditions of every day in the planning horizon D. This paper aims to cope with this problem of optimal toll design caused by the fluctuation of traffic flows, where the concept of robust optimization is taken for the modelling. On a particular day, each toll pattern τ can give rise to a corresponding TTC(τ). We first define the concept of *regret* for such a toll pattern on each day, which is the gap between the minimal TTC and TTC(τ). Then, a minimax model which minimizes the maximum regret on each day, is proposed for the robust optimal toll design. Note that, the minimal average TTC can also be taken as an alternative objective.

Since in the planning period D the network flow is fluctuating each day, it is difficult for the travelers to have an accurate prediction on the travel time. Thus, stochastic user equilibrium (SUE) is more suitable to capture their travel behaviors, compared with user equilibrium (Meng et al., 2014). In addition, for the optimal toll design considering SUE flows, it is more rational to take the stochastic system optimum (SSO) as the objective (Liu et al., 2014a), compared with the deterministic system optimum. Hence, in this paper we assume that the flow evolution process follows day-to-day dynamics under SUE constraints, and take travelers' expected total travel cost (ETTC) as the system wide index. However, formulating and solving day-to-day models or SUE/SSO models individually are known to be very challenging. The optimal design of distance-based tolls in a network considering stochastic day to day dynamics is thus difficult to address, which is still an open question in the literature and tackled in this paper.

1.1. Literature review

Due to the inequity of flat pricing patterns which undercharge long journeys and over-restrain short journeys (Meng et al., 2012), a distance-based pricing pattern was recommended by May and Milne (2000) as an alternative for flat toll patterns. In a distance-based congestion pricing scheme, the toll is levied in terms of the travel distance, either linearly (e.g., Mitchell et al., 2005; Namdeo and Mitchell, 2008) or nonlinearly (e.g., Wang et al., 2011; Lawphongpanich and Yin, 2012). Linear models assume that the toll is linearly proportional to the travel distance, making it easier for analysis due to the additivity of the toll charge. However, according to Lawphongpanich and Yin (2012), the actual congestion toll, in most cases, is nonlinear, i.e., the total charge for a trip cannot be proportionally divided to be the charges on its component links (Meng et al., 2012). For the distance-based toll charge function, no practical data could be collected for the analysis of a proper functional form or the calibration of such a function. Hence, it is proper to assume that it is generic to any positive and nonlinear function, which includes the fixed toll rate. Thus, this paper also adopts the nonlinear function form for the distance-based tolls.

A nonlinear pricing pattern known as the two-part tariff, which can be regarded as a special case of the piecewise linear toll scheme, was adopted by Lawphongpanich and Yin (2012) to study the nonlinear pricing on transportation networks. Meng et al. (2012) and Liu et al. (2014a) extended the piecewise linear toll scheme from only two linear intervals to multiple intervals. Sun et al. (2016) investigated the equity issues of distance-based tolls. However, all these formulations are based on static traffic assignment theory, either deterministic or stochastic. Recently, Daganzo and Lehe (2015) studied the distance-dependent, time-varying congestion pricing scheme based on the macroscopic fundamental diagram theory of traffic dynamics. However, this model is a within-day dynamic model, which cannot reflect the day-to-day flow evolution process after implementing a new toll pattern.

For the congestion toll design problem with day-to-day dynamics, Wie and Tobin (1998) solved it by formulating a convex control model of the dynamic system optimal traffic assignment on general traffic networks. Sandholm (2002) proposed a dynamic congestion pricing considering road users' learning behavior and day-to-day route choice adjustment process to guarantee an efficient utilization of the entire network. Thereafter, Friesz et al. (2004) studied the day-to-day dynamic toll with the objective of maximizing the net present value of social welfare and the constraint of a minimum revenue target. Yang et al. (2007) and Wang et al. (2015) considered the convergence speed and rapidity of restoring the normal state after disruption, respectively. More recently, Guo et al. (2015) proposed a concise and practical day-to-day dynamic pricing pattern based on Friesz et al. (2004) and Yang et al. (2007), and the tolls on each day were merely determined by the flows and tolls on the previous day. However, as claimed in Ye et al. (2015), all of these day-to-day dynamic toll patterns required either an explicit mechanism for road users' route choice adjustment process or adjustable tolls. Ye et al. (2015) studied