An eigenmodel for iterative line planning, timetabling and vehicle scheduling in public transportation ☆

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ABSTRACT

Planning a public transportation system is a multi-objective problem which includes among others line planning, timetabling, and vehicle scheduling. For each of these planning stages, models are known and advanced solution techniques exist. Some of the models focus on costs, others on passengers' convenience. Setting up a transportation system is usually done by optimizing each of these stages sequentially.

In this paper we argue that instead of optimizing each single step further and further it would be more beneficial to consider the whole process in an integrated way. To this end, we develop and discuss a generic, bi-objective model for integrating line planning, timetabling, and vehicle scheduling. We furthermore propose an *eigenmodel* which we apply for these three planning stages and show how it can be used for the design of iterative algorithms as heuristics for the integrated problem. The convergence of the resulting iterative approaches is analyzed from a theoretical point of view. Moreover, we propose an agenda for further research in this field.

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1. Planning in public transportation

The planning process in public transportation can be split into the following planning stages: After the design of the public transportation network, the lines have to be planned. This includes line planning and frequency setting. After that, the timetable can be designed, followed by vehicle scheduling, crew scheduling, and crew rostering. For railway transportation, even more planning stages have to be considered, e.g., platforming. For all of these planning problems, models are known and advanced solution techniques are available. However, going through all these stages sequentially leads only to suboptimal solutions. This motivates recent research on integrated planning in which two or even more of the planning stages are treated simultaneously. In this paper we present an iterative approach to integrated planning. We formulate this approach for the three consecutive planning stages line planning, timetabling, and vehicle scheduling. These three planning stages are a good starting point, since they involve cost- and passenger-oriented objective functions and they are a crucial part for the planning process in public transportation. Our setting is abstract enough to allow bus, tram, underground, and railway applications.

The outline of the paper is as follows. After a short description of line planning, timetabling and vehicle scheduling in Sections 2 we argue that the classic sequential planning process in which first the lines are fixed, then the timetable is designed, and then the vehicle schedules are optimized only leads to suboptimal solutions. In Section 3 we describe how an integrated model can be set up, in particular how the two different objective functions, to minimize the costs and to

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maximize the passengers' convenience, can be treated. In Section 4 we propose an algorithmic scheme, called *eigenmodel* which considers all three stages and can be used to construct and to improve solutions. We illustrate the eigenmodel in Section 5 by three different algorithms that can be derived from this scheme. The paper is concluded in Section 6 with an agenda for further research.

2. State of the art: sequential planning and first integrated approaches

In this section we briefly review the three planning stages line planning, timetabling and vehicle scheduling, and how they can be combined. There are many more aspects, in particular for rail transportation, which are not treated here in order to keep the methodology general. In the design of iterative algorithms as proposed in Section 4 such aspects can be included.

2.1. Line planning

Line planning is treated in many papers, starting with Patz (1925) in 1925 and is still a topic of ongoing research, see Schöbel (2012) for a rather recent survey. In contrast to other areas there is not one unique line planning model, but a variety of different models. As input data, always a Public Transportation Network (PTN) is given. It consists of a set of stations V connected by possible links E. A line l is a path through the PTN=(V, E), and its frequency specifies how often a service is offered (within the planning period) along this path.

As input data, most models require a line pool, i.e., a set of pre-specified lines $\mathcal{L} = \{l_1, \ldots, l_p\}$ from which the line plan is to be chosen. Formally, a *line concept* is a vector $x^L \in \mathbb{N}^p$ where x_l^L specifies the frequency of line $l \in \mathcal{L}$.

Dependent on the respective model chosen, there exist different constraints for feasibility of a line plan and its frequencies. A basic constraint included in nearly all line planning models is to require that the total number of trains operated on every edge $e \in E$ of the PTN lies between a given lower and a given upper edge frequency $f_e^{\min}$ and $f_e^{\max}$, see (1). Several objective functions have been proposed, among them to minimize the costs (Claessens et al., 1998), to maximize the number of direct travelers (Bussieck et al., 1996), to minimize the riding times (Borndörfer et al., 2007), the traveling times (including a penalty for transfers) (Schöbel and Scholl, 2006), or the number of transfers (Harbering, 2016). The latter four are examples for passenger-oriented objective functions. Currently, different stopping patterns along the same network (Bull et al., 2016, 2015; Burggraeve et al., 2016) and the automatic generation of line pools (Gattermann et al., 2016c) are topics of ongoing research in line planning.

For the integration of line planning and timetabling later on, it is important how passengers are treated in line planning models. In order to consider the passengers in the line planning stage, there are (at least) two possibilities: Demand data may be given as traffic loads w_e for every edge $e \in E$, specifying the number of passengers that wish to travel along edge e, or as OD-data w_{st} for any pair of stations $s, t \in V$ specifying the number of passengers who wish to travel between s and t. While line planning based on traffic loads is rather unrealistic since the passengers' paths strongly depend on the line concept, models using OD-data are unfortunately still too hard to be solved from a computational point of view.

2.2. Timetabling

In order to set up a timetable one uses the given lines and specifies their resulting *events* $\mathcal{E}$, namely every arrival and departure of a vehicle at a station. Basically, a timetable x^T specifies a time x_j^T for every event j. Timetables may be aperiodic or periodic if events are repeated every P time units. If a periodic timetable is searched, $\mathcal{E}$ is usually defined as the arrival and departure of every *line* at every station, since the vehicles' arrival and departure times are then periodically repeated. Events are connected by activities: driving activities link the departure of a line at some station with its arrival at the next station, and waiting (or dwell) activities are between the arrival of a line at a station and its departure at the same station. Activities linking events from different lines may describe transfers of passengers or headway constraints. The resulting graph $(\mathcal{E}, \mathcal{A})$ is called event-activity network. Every activity comes with a time window, and it is required that the real duration of the activity which is then given by the timetable lies in this time window.

There are two streams of papers on timetabling: There is a group of railway oriented timetabling papers which mainly consider the capacity of the track system. Here, timetabling is treated together with or after a routing step (Lusby et al., 2011) since the specific paths of the trains through the different track segments are not yet specified by the lines. If the routes of the trains are fixed, the resulting problem can be treated as a resource-constraint project-scheduling problem (Mascisa and Pacciarelli, 2002). Possible objective functions in these papers are to find a timetable which is as close as possible to a reference timetable (Caprara et al., 2002), to minimize the driving times of the trains, or to maximize the number of trains which can be scheduled through the system, see Törnquist (2005) for an overview. Methods include branch and bound (D'Ariano et al., 2007).

In our paper we deal with another group of timetabling papers which consider the passengers and aim at minimizing their traveling times. To this end, it is assumed that it is already known how many passengers travel with every train (or line) and how many passengers wish to transfer between two different lines meeting at the same station. Capacity constraints on the tracks are approximated by so-called headway activities which require a minimal time to be passed between two consecutive trains. They can also be used to ensure secure traffic on single-track lines. The basic model used for finding a