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Group decision making based on linguistic distributions and hesitant assessments: Maximizing the support degree with an accuracy constraint

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ABSTRACT

The hesitant fuzzy linguistic term set (HFLTS) and the linguistic distribution are becoming popular tools to model linguistic expressions with multiple linguistic terms in decision problems. Compared with HFLTSs, linguistic distributions provide more probabilistic preference information over linguistic terms, and are useful to express decision makers' preferences accurately. However, in a group decision context a linguistic distribution based group opinion will bring great difficulty for the group to take an accurate action. Meanwhile, the linguistic group opinion should obtain enough support from decision makers in the group. To tackle these issues, based on the use of linguistic distributions and HFLTSs we propose a new linguistic group decision model called the maximum support degree model (MSDM), aiming at maximizing the support degree of the group opinion as well as guarantying the accuracy of the group opinion. A mixed 0–1 linear programming approach is presented to solve the MSDM, and a feedback adjustment is employed to improve the support degree of the group opinion. Finally, the use of the MSDM in multiple attribute group decision making is demonstrated.

1. Introduction

Linguistic decision making is very common in day-to-day activities due to its convenience and naturalness in directly expressing human beings' opinions through linguistic information [17]. When solving decision making problems with linguistic information, it is very necessary to study computing with words (CWW) [22,25,26,31,41,42]. One representative CWW model is the 2-tupe linguistic representation model [20], which avoids the computation weakness in information loss and expresses linguistic information in a more precise way with a wide range of applications (e.g., [3,12,13,23,35,37]). Based on the 2-tuple linguistic model, different developments have been made such as the linguistic hierarchy model [18,19], the proportional 2-tuple linguistic model [38] and the numerical scale model [7,10].

However, the models mentioned above mainly focus on the situations that information is elicited in the form of a single term, which is quite limited in problems defined under uncertainty. Due to the limitation of the given information or the vague and imprecise knowledge of decision makers, it is convenient for decision makers to consider using multiple linguistic terms [1,11,31,33] to express their opinions. To overcome the mentioned limitation, two kinds of linguistic expressions with multiple linguistic terms can be summarized:

(1) The hesitant fuzzy linguistic term set (HFLTS), which aims at using several consecutive terms to elicit linguistic information. Rodríguez et al. [32] put forward the concept of HFLTS. Beg and Rashid [2] proposed a new method to aggregate HFLTSs based opinions of decision makers on different criteria. Liu and Rodríguez [24] proposed a representation of HFLTSs by means of a fuzzy envelope to carry out the CWW processes. Wei et al. [39] introduced the aggregation operators and comparisons of HFLTSs. Dong et al. [5,7] presented a novel approach to deal with consensus reaching process with hesitant linguistic assessments in group decision making (GDM) and developed the unbalanced HFLTS model, respectively. Zhang and Guo [46] proposed new operations for the HFLTS with applications in multi-attribute group decision making (MAGDM). The recent progress of the HFLTS in

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decision making can be found in the position paper (see Rodríguez et al. [30]).

(2) *The linguistic distribution*: Zhang et al. [44] proposed the concept of linguistic distributions over a linguistic term set. Dong et al. [9] introduced a methodology to deal with unbalanced linguistic distributions with interval symbolic proportions under multi-granular contexts. Zhang et al. [45] introduced a new computational model to operate with multi-granular linguistic distributions in large-scale MAGDM. Wu and Xu [40] and Chen et al. [4] proposed some connections between HFLTSs and linguistic distributions, and presented new approaches based on possibility distributions to address MAGDM with hesitant fuzzy linguistic information.

In GDM problems with uncertainty (uncertain decision contexts or vague knowledge of decision makers), it is natural and better for decision makers to use linguistic expressions with multiple terms such as HFLTSs and linguistic distributions to express their opinions. Both HFLTSs and linguistic distributions can make decision makers' expressions more flexible. But compared with HFLTSs, linguistic distributions provide more probabilistic preference information over linguistic terms, and are useful to express decision makers' preferences accurately. On the other hand, when multiple decision makers are involved in a GDM problem, a linguistic distribution is not a suitable choice as a group opinion because:

(1) When decision makers' individual opinions expressed by linguistic distributions are directly aggregated into a group opinion, this group opinion will be a linguistic distribution. Generally, based on this kind of group opinion, it will be difficult for decision makers to take an accurate group action. Thus, we hope the group opinion is the one with some accuracy (e.g., an HFLTS).

(2) We hope that a group opinion can obtain enough support from decision makers in the group. So we need to measure the degree of the support for a group opinion, and find out the group opinion to maximize its support degree in the aggregation process. Particularly, when the support degree of the group opinion is not enough (e.g., does not reach an established level), proper feedback processes will be employed to improve the support degree among the group.

Therefore, a challenge for analysts is how to obtain a group opinion with some accuracy as well as maximize the support degree of the group opinion in GDM problems with uncertainty based on linguistic expressions such as linguistic distributions and HFLTSs.

Inspired by the idea, a new linguistic group decision approach called the maximum support degree model (MSDM) will be developed to tackle the proposed problem, and the rest of this paper is organized as follows. Section 2 presents the preliminaries regarding the 2-tuple linguistic model, HFLTS and linguistic distributions. Then, Section 3 proposes the problem to be studied and develops the MSDM. Next, Section 4 introduces procedures to solve the MSDM. Furthermore, Section 5 puts forward a feedback adjustment process to improve the support degree of the group opinion. In addition, Section 6 discusses the use of the MSDM in MAGDM. Finally, conclusions and future research are included in Section 7.

2. Background

In this section, we introduce the basic knowledge regarding the 2-tuple linguistic model, HFLTS and linguistic distributions.

2.1. The 2-tuple linguistic model

Let $L = \{l_0, l_1, \ldots, l_g\}$ be a linguistic term set with odd cardinality, and $g+1$ be the cardinality of L. L should satisfy the following characteristics [20,25]:

(1) The set is ordered: $l_t \geq l_s$ if $t \geq s$;
(2) There is a negation operator: $Neg(l_t) = l_s$ such that $t + s = g$.

The term l_t $(t = 0, 1, \ldots, g)$ represents a possible value for a linguistic variable, and the basic notations and operation laws of linguistic variables are introduced in [20].

Herrera and Martínez [20] proposed the 2-tuple linguistic model, in which linguistic information is represented by a linguistic 2-tuple (l_t, α), where $l_t \in L$ and $\alpha \in [-0.5, 0.5)$. The transformation function between linguistic 2-tuples and numerical numbers is defined below.

Definition 1. [20]: Let $L = \{l_0, l_1, \ldots, l_g\}$ be as before and $\beta \in [0, g]$ be a value representing the result of a symbolic aggregation operation. A linguistic 2-tuple that expresses the equivalent information to β is obtained by the function Δ:

$$\Delta : [0, g] \to L \times [-0.5, 0.5) \tag{1}$$

where $\Delta(\beta) = (l_t, \alpha)$ with $\begin{cases} l_t, t = round(\beta) \\ \alpha = \beta - t, \alpha \in [-0.5, 0.5) \end{cases}$, with *round* being the usual *rounding* operation. The set of all linguistic 2-tuples is denoted by $\bar{L}$, i.e., $\bar{L} = \{(l_t, \alpha) | l_t \in L, \alpha \in [-0.5, 0.5), t = 0, 1, \ldots, g\}$.

Clearly, Δ is a one to one mapping function and the inverse function of Δ is:

$$\Delta^{-1} : \bar{L} \to [0, g] \tag{2}$$

with $\Delta^{-1}(l_t, \alpha) = t + \alpha$.

For any 2-tuple of $\bar{L}$, there are the following computational operations:

(1) Negation operation: $Neg(l_t, \alpha) = \Delta(g - (\Delta^{-1}(l_t, \alpha)))$.
(2) Comparison operation:

Let (l_t, α_1) and (l_s, α_2) be two linguistic 2-tuples.

(a) If $t < s$, then (l_t, α_1) is smaller than (l_s, α_2);
(b) If $t = s$,
(i) $\alpha_1 = \alpha_2$, then (l_t, α_1) and (l_s, α_2) represent the same information;
(ii) $\alpha_1 < \alpha_2$, then (l_t, α_1) is smaller than (l_s, α_2).

Several aggregation operators such as the weighted averaging (WA) operator and the ordered weighted averaging (OWA) operator have been developed (see [20]). The details for the 2-tuple linguistic model can be found in Herrera and Martínez [20].

2.2. HFLTS

The 2-tuple linguistic model proposed by Herrera and Martínez [20] can deal with linguistic information with single terms, such as $(l_t, 0.4)$ for the case when a decision maker's preference is between l_t and l_{t+1}. However, there are situations that single terms cannot handle. To overcome the limitations, Rodríguez et al. [32] proposed the concept of HFLTS in which multiple consecutive terms such as $\{l_t, l_{t+1}, l_{t+2}\}$ are allowed to represent a decision maker's hesitant preference between l_t and l_{t+2}. The concepts of HFLTS and its envelope are introduced as Definitions 2 and 3.

Definition 2. [32]: Let $L = \{l_0, l_1, \ldots, l_g\}$ be a linguistic term set, and an HFLTS, denoted as H_L, is an ordered finite subset of the consecutive linguistic terms of L.