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Evidential reasoning with discrete belief structures

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ABSTRACT

In real-world applications, apart from being precise-valued or interval-valued, belief structure measurements can also be discrete-valued. However, problems relating to the combination of discrete-valued belief structures have not been resolved. Therefore, in the research presented in this paper, we explore the counterintuitive behavior associated with the combination of discrete evidence and extend the concept of evidential reasoning (ER) to evidential reasoning with a discrete structure in order to serve as the theoretical basis and as technical support for the fusion of discrete information. This method offers an approach to the normalization of discrete evidence, provides a means of objectively determining the weight of discrete evidence, and optimizes the combination of discrete evidence based on evidential reasoning. The results of various examples show that the method not only offers an effective solution to the combination of non-conflicting discrete evidence, but it also overcomes the counterintuitive results of combining internally or externally conflicting evidence.

1. Introduction

Evidence theory [1,2] was originally developed by Dempster and was later extended and refined by Shafer. The term is thus often referred to as the Dempster–Shafer (DS) theory of evidence. DS theory is a powerful and flexible mathematical tool for processing imprecise and uncertain information. It has been employed in a wide range of areas, including expert systems [3], uncertainty reasoning [4], pattern classification [5], fault diagnosis and detection [6], information fusion [7], multiple attribute decision analysis [8], image processing [9], regression analysis [10], risk analysis [11], e-commerce security [12], and water distribution systems [13].

Based on both decision theory and DS theory, an evidential reasoning (ER) approach was proposed by Yang and Singh [14] for multiple-attribute decision making (MADM) in the presence of uncertainty. In the past two decades, extensive research has been conducted on the ER approach. Initially, it was adopted by Yang [15], who proposed a model to address a wide range of MADM problems involving precise data, random numbers, and subjective judgments. Yang and Xu [16,17] later showed that the nonlinear features of the ER approach can be used to separate the unassigned belief degree into two parts: one relating to incompleteness and the other relating to the relative weight of each attribute. Further research on the ER algorithm was conducted by Huynh et al. [18], who proposed a general scheme of attribute aggregation that satisfies the synthesis axioms proposed by Yang and Xu [16,17]. To solve MADM problems with both probabilistic and fuzzy uncertainties, Yang et al. [19] explicitly re-analyzed ER in terms of DS theory to further develop this approach. Specifically, they modeled precise data, ignorance, and fuzziness under the unified framework of a distributed fuzzy belief structure, thereby obtaining a fuzzy belief decision matrix. In an attempt to solve both the interval uncertainty and fuzzy beliefs in assessing the alternatives of an attribute, Guo et al. [20] proposed a fuzzy interval grade ER, whereas Wang et al. [21] developed an ER approach with interval belief degrees for MADM. Hu et al. [22] sought to attain a dynamic fusion and proposed dynamic evidence reasoning. Their approach considers the time effect by introducing a belief decay factor. This factor reflects the idea that the credibility of evidence decreases over time. Recently, researchers have become increasingly interested in the ER approach. Yang and Xu [23] established a unique ER theory that can be used to combine multiple pieces of independent evidence according to their weight and reliability. Fu and Chin [24] thoroughly investigated a robust evidential reasoning approach that can be used to compare alternatives by measuring their robustness with respect to attribute weights in the ER context. In addition, the ER approach and its extensions have been widely applied to MADM problems in business performance assessment [25–30], environmental impact assessment [31], organizational self-assessment [32], safety analysis [33,34], bridge condition assessment [35], behavior prediction [36], fault prediction

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Table 1
Results of different combination methods.

Methods		The bbas of combined evidence
Dempster [1]		$m(\{x\})$=0.0000, $m(\{y\})$=0.0080, $m(\{z\})$=0.9920
Martin [64]		$m(\{x\})$=0.9261, $m(\{y\})$=0.0013, $m(\{z\})$=0.0652, $m(\{x, z\})$=0.0074
Murphy [65]		$m(\{x\})$=0.4600, $m(\{y\})$=0.1900, $m(\{z\})$=0.0700, $m(\{x, z\})$=0.2200, $m(\{y, z\})$=0.0600
Chen [71]		$m(\{x\})$=0.834, $m(\{y\})$=0.022, $m(\{z\})$=0.060, $m(\{x, z\})$=0.069, $m(\{y, z\})$=0.002, $m(\{\Theta\})$=0.012
Yu [72]		$m(\{x\})$=0.9716, $m(\{y\})$=0.0004, $m(\{z\})$=0.198, $m(\{x, z\})$=0.0082
Yang [23]		$m(\{x\})$=0.5338, $m(\{y\})$=0.1515, $m(\{z\})$=0.1213, $m(\{x, z\})$=0.0436, $m(\{y, z\})$=0.1498
Chen [70]	$\varepsilon_1 = 0.0$	$m(\{x\})$=0.5326, $m(\{y\})$=0.1519, $m(\{z\})$=0.1195, $m(\{x, z\})$=0.0435, $m(\{y, z\})$=0.1525
	$\varepsilon_2 = 0.5$	$m(\{x\})$=0.3842, $m(\{y\})$=0.3015, $m(\{z\})$=0.1100, $m(\{x, z\})$=0., $m(\{y, z\})$=0.0600
	$\varepsilon_3 = 1.0$	$m(\{x\})$=0.4600, $m(\{y\})$=0.1900, $m(\{z\})$=0.0700, $m(\{x, z\})$=0.2200, $m(\{y, z\})$=0.0600
Zhang [73]		–
Proposed		$m(\{x\})$=0.6090, $m(\{y\})$=0.0989, $m(\{z\})$=0.0728, $m(\{x, z\})$=0.2025, $m(\{y, z\})$=0.0167

Table 2
Comparison of combination results.

	Methods	The bbas of evidence		
		$\{x\}$	$\{y\}$	$\{z\}$
Group 1	DRDE	0.033→0.246	0.258→0.739	0.101→0.522
	Proposed method	0.235→0. 313	0.324→0.430	0.240→0.382
Group 2	DRDE	0.000→1.000	0.000→1.000	0.000→1.000
	Proposed method	0.445→0.537	0.006→0.023	0.500→0.536
Group 3	DRDE	0.000→0.996	0.000→0.049	0.004→1.000
	Proposed method	0.406→0. 430	0.236→0.245	0.334→0.351
Group 4	DRDE	0.000→1.000	0.000→0.498	0.000→1.000
	Proposed method	0.872→0.980	0.015→0.023	0.006→0.105

[37], risk analysis [38], job offering [39], software selection [40], and group decision analysis [41].

The original DS theory or ER approach and its extensions mentioned above were developed to process deterministic evidence. These methods require all probability masses assigned to focal elements to be precise. However, in many circumstances, owing to the inability of humans to provide a complete judgment or on account of the lack of information, some or all probability masses given by decision-makers (DMs) may be uncertain or otherwise imprecise. Therefore, it is necessary to modify the original theory. To date, several attempts have been made to extend the DS theory or ER approach to interval belief structures [42–47]. These methods are used to solve the problem associated with the combination of continuous interval belief structures. Wang et al. [47] aimed to overcome the shortcomings of existing methods [42–46] by investigating the issues surrounding the combination and normalization of interval-valued belief structures and developed a new logically correct optimality approach. However, because of the difference of the data form between discrete values and interval values, the models that are used to optimize the combination and normalization for interval-valued belief structures are not applicable to discrete belief structures (also known as discrete evidence). In addition, Wang et al. [47] did not provide a means of objectively determining the weight of discrete evidence. To address these issues, we herein propose an approach we refer to as evidential reasoning with discrete belief structures.

Further research relating to evidential reasoning with discrete belief structures is therefore necessary. In many decision situations, assessment information measurements can be either precise-valued or interval-valued, but they can also be discrete-valued. Given a set of alternatives, although the DM may be unable to provide precise ordinal data, they may be able to offer discrete ordinal data, if they are not absolutely sure about their estimation. For example, a buyer plans to purchase a house and there are six types of candidate houses (H001, H002, H003, H004, H005, and H006). The DM considers that H002 is ranked among the top three, where "top three" is a discrete preference ordinal {1, 2, 3}. Because this kind of discrete data is ubiquitous, it is necessary to solve the problem of discrete information fusion. Existing approaches [48–53] have made significant contributions to solving the fusion problems with discrete information on alternatives. However, these approaches cannot accommodate discrete information on uncertain candidate schemes, but only on certain candidate schemes. In fact, due to limited cognition or the complexity of decision-making problems, the DM often can confirm neither the preference ordinals nor the candidate schemes. In the above example, if one DM considers that H002 or H003 is ranked {1, 2, 3}, then it means that the preference ordinal {1, 2, 3} is uncertain and the candidate scheme is an element of the set {H002, H003}, even though they are uncertain as to which one it is. In this case, existing decision-making approaches are invalid. Evidential reasoning makes it possible to assign basic probabilities not only to single propositions but also to any of their subsets, thereby allowing candidate schemes to be uncertain. Besides, previous research [10] has shown that the discrete preference ordinal can be converted into equivalent discrete belief values. Hence, research on evidential reasoning with discrete belief structures is worthy of attention.

The aim of this paper is to extend the concept of evidential reasoning (ER) to evidential reasoning with a discrete structure in order to serve as the theoretical basis and as technical support for the fusion of discrete information. First, we explore the counterintuitive behavior of combining discrete evidence and offer an approach to the normalization of discrete evidence. Then, a means of objectively determining the weight of this discrete evidence is provided. Furthermore, an optimization model is built to combine discrete belief structures. Finally, the feasibility and validity are proved through illustrative examples.

The remainder of this paper is organized as follows. Section 2 introduces the relevant concepts of DS theory and explores the counterintuitive behavior of discrete evidence combination. Section 3 surveys evidential reasoning and discusses its advantages. In Section 4, the normalization of discrete belief structures is described. In addition, we propose a solution to determine the weight of discrete evidence and present an optimal combina-