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Operations and integrations of probabilistic hesitant fuzzy information in decision making

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ABSTRACT

In recent years, the hesitant fuzzy set (HFS) has been widely applied in practical decision making processes. As a matter of fact, it indeed describes the thoughts of experts better because of a better tolerance. But there is a big defect in its applications—the serious loss of information. To improve the HFS, the probabilistic hesitant fuzzy set (P-HFS) has been put forward. It adds the probability to the HFS and can retain more information than the HFS. However, the P-HFS is still not perfect. In this paper, the definitions of the P-HFS and the probabilistic hesitant fuzzy element (P-HFE) are improved at first. After that, the properties and operations of the improved P-HFEs are studied, and some corresponding aggregation operators are given. Furthermore, the concept of P-HFEs with the continuous form is proposed, some operations and distance measures for them are given. Finally, the integrations of the improved P-HFEs are used to deal with a practical case concerning the automotive industry safety evaluation.

1. Introduction

Since Zadeh [1] introduced the fuzzy set in 1965, many scholars have studied some extended forms of fuzzy set, such as intuitionistic fuzzy set (IFS) [2,3], type-2 fuzzy set [4], type-n fuzzy set [4], fuzzy multiset [5], and hesitant fuzzy set (HFS) [6]. Among them, the HFS [6] has been widely applied in practical decision making processes. The core of the HFS is the hesitant fuzzy element (HFE) [7]. Since its inception, some researchers have done lots of work to develop the HFS theory [8–18]. These works make the decision making methods more convenient and reliable. But, with the deepening of the research, a defect of the HFE, which cannot be ignored, is gradually showed up—the serious loss of information. Some scholars have tried to solve the problem in recent years. Bedregal et al. [19,20] tried to use the fuzzy multisets to improve the HFSs. To a certain degree, the method solved the problem well. But, there still exist some problems which are very difficult to solve. For example: (1) The sum of all frequency of occurrence of all different memberships in an element cannot be less than 1; (2) The frequency of occurrence of every membership in an element cannot be irrational numbers; (3) The expression of fuzzy multisets is too complex to use. In 2014, Zhu [21] tried to add the probability into the HFS. This original research can overcome the defect of HFSs and HFEs to a great extent and solve the problems of the fuzzy multisets effectively. He named the new HFS probabilistic hesitant fuzzy set (P-HFS). Then, some scholars have carried on further research to the P-HFS. Zhang and Wu [22] studied some operations of P-HFSs and gave their application to multi-criteria decision making. All of those researches make the study of P-HFS reach a new height. We will improve and further perfect the P-HFS theory in this paper, and there are two main innovation points in our work:

1) The conditions of the P-HFS and the P-HFE are improved, and thus, the decision makers (DMs) can be given more space of hesitation. In recent several years, all of the previous studies of P-HFSs are based on the condition that the sum of all probabilities in a P-HFE is 1. In fact, this condition is not necessary. When a DM describes his/her thought with P-HFEs, he/she can keep a certain amount of hesitation. In other words, the condition that the sum of all probabilities in one P-HFE is smaller than 1 should also be allowed.

Example 1. Suppose that a consumer wants to buy an apartment in Wanda Plaza in Chengdu, China. He/she mainly focuses on the house type and asks an expert for advice. In the case that the full mark is 100 points, the expert is 60% sure that the house type scores of the apartment in Wanda Plaza could be 70, and he/she is 30% sure that the scores could be 80. So he/she gives his suggestion by the form: $House\ Type\ (Wanda\ Plaza) = \{0.7(0.6), 0.8(0.3)\}$.

Example 2. Suppose that an online store sells a kind of apples, and there are 100 evaluations of the sweetness for this kind of apples in the store. Fifty of them give eighty points, twenty-five of

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them give seventy-five points, twenty of them give seventy points and five of them do not show their perspectives. In this case, the information we have gotten can be represented by the form: $Sweetness = \{70(0.2), 75(0.25), 80(0.5)\}$.

Therefore, it can be seen from the above two examples that, sometimes, the complete probabilistic information of P-HFEs cannot be obtained. In a decision making problem, the DMs may have a certain degree of hesitation. That is to say, to a certain extent, the DMs cannot make a certain decision. If we force them to give the complete information, the uncertain information may lead to erroneous decision results. In a group decision making problem, like Example 2, some people cannot give their advices. Thus, under the situation like this, the complete probability information cannot be gotten, and then, the original P-HFS fails totally. However, our improved P-HFS can solve the problem effectively.

In decision making problems, if there exists a little deviation on the decision making information given by DMs, there may be a huge error in the result. So, we must increase the error-tolerant rate of our decision making tools. The fuzzy set theory is developed for this and the same to its extended forms. The advantages and disadvantages of the HFS can be seen intuitively, the original P-HFS and our improved P-HFS in the following table:

	Make the DMs have more choices	Retain most decision making information	Give the DMs more space to hesitate
The HFS	√	×	×
The original P-HFS	√	√	×
The improved P-HFS	√	√	√

2) The P-HFE with the continuous form is proposed. Through a suitable use of density functions, this concept makes the discrete P-HFEs become continuous, and this work also extends the coverage of P-HFEs. Sometimes, the P-HFEs with the continuous form may express the DMs' thoughts better. In addition, the situation of lack of decision making information may be encountered in many times. Benefited from P-HFEs with the continuous form, we can use the form of, for example, a normal distribution to estimate the lost information, and such estimations can be more reasonable.

The structure of this paper is organized as follows: In Section 2, the conditions of the P-HFS and the P-HFE are improved at first. Based on which, the properties and operations of the improved P-HFEs are studied, some corresponding aggregation operators for them are given. Section 3 presents the P-HFEs with the continuous form and studies their operations and distance measures. Section 4 makes a practical case study concerning the automotive industry safety evaluation, which is a multi-attribute decision making problem based on the integration of the proposed P-HFEs. The paper ends with a conclusion in Section 5.

2. Probabilistic hesitant fuzzy elements

2.1. The concept of probabilistic hesitant fuzzy elements

In 2014, Zhu [21] defined the concept of the P-HFS:

Definition 1. [21]**.** Let X be a fixed set, a P-HFS on X is in terms of a function that when applied to X returns to a subset of [0, 1], which is expressed as $H = \{\langle x, h_x(p_x)\rangle | x \in X\}$, where both h_x and p_x are two sets of some values in [0, 1]. h_x denotes the possible membership degrees of the element $x \in X$ to the set E; p_x is a set of probabilities associated with h_x. $h_x(p_x)$ is called the probabilistic hesitant fuzzy element (P-HFE).

For the sake of more convenient application and more concise description, we denote the P-HFE $h_x(p_x)$ as $h(p)$, and a generic P-HFE is expressed as $h(p) = \{\gamma_l(p_l) | l = 1, 2, \cdots, |h(p)|\}$, where p_l is the probability of the membership degree γ_l, $\gamma_l(p_l)$ is called a term of the P-HFE, $|h(p)|$ is the number of all different membership degrees, and $\sum_{l=1}^{|h(p)|} p_l = 1$.

Obviously, the P-HFE can retain much more evaluation information from the DMs than the HFE, and using P-HFE instead of HFE to express the DMs' preferences is more reliable and reasonable. But one of the above conditions is interesting. Inspired by Pang et al. [23], we now discuss the condition $\sum_{l=1}^{|h(p)|} p_l = 1$, from which the complement information of discrete probabilistic distribution of all possible membership degrees can be gotten. In many practical applications, some DMs cannot provide complement information because, for example, they do not have enough capacity. Furthermore, if there is not only one DM, there exists a situation that not all the DMs can give their information. In these cases, partial ignorance exists, and we have $\sum_{l=1}^{|h(p)|} p_l < 1$. Particularly, if $\sum_{l=1}^{|h(p)|} p_l = 0$, it means complete ignorance. Based on the discussion above, we try to modify Definition 1 in the following note:

Note. In Definition 1, if change the condition $\sum_{l=1}^{|h(p)|} p_l = 1$ to $\sum_{l=1}^{|h(p)|} p_l \le 1$, then, a new kind of P-HFEs can be obtained, called Weak P-HFEs.

According to the discussion above, the P-HFEs are a special case of the weak P-HFEs. In the remainder of this paper, the term P-HFE must be understood as weak P-HFE.

Example 3. If five DMs give their evaluation values, for example, {0.7, 0.8, 0.8, 0.9, 0.9} for an alternative respectively, then, the evaluation values as some possible membership degrees in a P-HFE can be considered. If the weight of each DM is same, then, a P-HFE can be obtained: $h(p) = \{0.7(0.2), 0.8(0.4), 0.9(0.4)\}$. Using the HFE to express the evaluation information from the DMs, the HFE will be $h = \{0.7, 0.8, 0.9\}$.

However, if the fifth DM does not give his/her evaluation value because of his lack of knowledge, then the evaluation values are changed to {0.7, 0.8, 0.8, 0.9}. Moreover, the above P-HFE $h(p) = \{0.7(0.2), 0.8(0.4), 0.9(0.4)\}$ is changed to $h'(p) = \{0.7(0.2), 0.8(0.4), 0.9(0.2)\}$. In this case, the sum of probabilities in $h'(p)$ is less than 1.

As we all know, the positions in a HFE can be moved arbitrarily. As a result, in the following, we propose the concept of the ordered P-HFE:

Definition 2. Given a P-HFE $h(p) = \{\gamma_l(p_l) | l = 1, 2, \cdots, |h(p)|\}$, it is called an ordered P-HFE, if its terms $\gamma_l(p_l),\ l = 1, 2, \cdots, |h(p)|$ are lined up based on the values of $\gamma_l \cdot p_l,\ \ l = 1, 2, \cdots, |h(p)|$ in descending order.

Example 4. Considering the P-HFE $h(p) = \{0.7(0.25), 0.8(0.5), 0.9(0.25)\}$ in Example 3, if we line up its terms according to the value $\gamma_l \cdot p_l$, then a new P-HFE can be obtained. Since $0.7 \times 0.25 = 0.175$, $0.8 \times 0.5 = 0.4$, and $0.9 \times 0.25 = 0.225$, then the new P-HFE is $h'(p) = \{0.8(0.5), 0.9(0.25), 0.7(0.25)\}$. Thus, the new P-HFE is an ordered P-HFE.

The ordered P-HFEs are better visualized than the general P-HFE, and they are easier to be used in operations. With Definition 2, below a method is given to judge whether two P-HFEs are equal to each other.

Let $h_1(p)$ and $h_2(p)$ be two ordered P-HFEs, where $h_1(p) = \{\gamma_{1_l}(p_{1_l}) | l = 1, 2, \cdots, |h_1(p)|\}$ and $h_2(p) = \{\gamma_{2_l}(p_{2_l}) | l = 1, 2, \cdots, |h_2(p)|\}$. If the conditions: (1) $|h_1(p)| = |h_2(p)|$; (2) $\gamma_{1_l} = \gamma_{2_l}$ and $p_{1_l} = p_{2_l}$ $(l = 1, 2, \cdots, |h_1(p)|)$ hold, then we say that $h_1(p)$ equals $h_2(p)$, denoted as $h_1(p) = h_2(p)$.

Given some general P-HFEs, they can be changed to the ordered P-HFEs by Definition 2, and then, the above method can be used to analyze them.