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Blind compressive sensing using block sparsity and nonlocal low-rank priors ☆

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ABSTRACT

Without knowing the sparsity basis, Blind Compressive Sensing (BCS) can achieve similar results with those Compressive Sensing (CS) methods which rely on prior knowledge of the sparsity basis. However, BCS still suffers from two problems. First, compared with block-based sparsity, the global image sparsity ignores the local image features and BCS approaches based on it cannot obtain the competitive results. Second, since BCS only exploits the weaker sparsity prior than CS, the sampling rate required by BCS is still very high in practice. In this paper, we firstly propose a novel blind compressive sensing method based on block sparsity and nonlocal low-rank priors (BCS-BSNLR) to further reduce the sampling rate. In addition, we take alternating direction method of multipliers to solve the resulting optimization problem. Experimental results have demonstrated that the proposed algorithm can significantly reduce the sampling rate without sacrificing the quality of the reconstructed image.

1. Introduction

The theory of Compressive Sensing (CS) [1–3] has drawn quite an amount of attention in digital signal processing. Different from the traditional methodology of sampling followed by compression, CS conducts sampling and compression at the same time. From much fewer measurements than suggested by the Nyquist sampling theory, CS also states that an image can be recovered from under-sampled measurements when it is sparse in some domain.

The goal of CS is to reconstruct a signal $x \in \mathbb{R}^m$ from measurements $b = Ax$, where $A \in \mathbb{R}^{n \times m}$ is the sensing matrix and $n \ll m$. This problem is ill-posed in general and therefore has infinitely many possible solutions. In CS we seek the sparest vector s that satisfies $x = Ps$:

$$s = \arg\min \|s\|_0 \quad s.t. \quad b = APs \tag{1}$$

where $\|\cdot\|$ is the l_0 norm which counts the number of nonzero elements of the vector and P is the sparsity basis.

The principle of CS relies on the fact that the signal has a sparse representation in a given sparsity basis P [4–6] that is universal for the considered signal class of interest. However, such universal sparsity basis does not necessarily result in sparsest representation, which is crucial for the successful recovery. Moreover, we cannot know the sparsity basis of the unknown signal. Due to this, in [7] the concept of Blind Compressive Sensing (BCS) has been introduced, which aims at simultaneously learning the sparsity basis and reconstructing the signal/image. The only useful prior in BCS is that there exists some basis in which the original signal x is sparse. It has been demonstrated that, without knowing the sparsity basis, BCS can achieve similar results with those CS methods which rely on the prior knowledge of the sparsity basis.

However, BCS still suffers from two problems when it is applied to images. First, compared with block-based sparsity, the global image sparsity ignores the local image features and BCS approaches based on it cannot obtain the competitive reconstruction results. Second, BCS only exploits the weaker prior and does not take use of other useful priors, the sampling rate on an image required by BCS is still very high in practice.

More recently, the concept of sparsity in CS has evolved into various sophisticated forms including model-based or Bayesian, nonlocal sparsity [8–11], structured sparsity and group sparsity [12–14], where exploiting high-order dependency among sparse coefficients has shown beneficial to CS image recovery. Therefore, it is desirable to pursue more efficient solutions that can exploit the benefits of both BCS and high-order dependency among sparse coefficients.

Motivated by low-rank regularization based approaches which can exploit high-order dependency among similar blocks and have shown the state-of-the-art performance in compressive sensing

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[15], face recognition [16–18], image denoising [19] and saliency detection [20]. In this paper, we take use of block sparsity and nonlocal low-rank priors simultaneously for blind compressive sensing image recovery (BCS-BSNLR). Compared with traditional BCS approaches, the contributions of our proposed BCS-BSNLR are mainly two-folded. First, BCS-NLSR explicitly and effectively characterizes the intrinsic block sparisty and nonlocal low-rank property of natural image simultaneously in a unified framework, which can help to reduce the sampling rate without sacrificing the quality of the reconstructed image. It is the first algorithm to do this in BCS works. Second, the images are divided into overlapping blocks rather than non-overlapping blocks, reducing artifacts in reconstructed images.

Related work. Some research works have been done on blind compressive sensing. But the algorithms they proposed are significantly different from our BCS-BSNLR. Refs. [7,21,22] all exploited the whole image sparsity in blind compressive sensing. Since learning sparsity basis P from compressive measurements did not admit a unique solution, Refs. [7,23] imposed different additional structure on P or x. This is quite different from our BCS-BSNLR, which exploits block-based sparsity that can capture the local image features effectively. In addition, we characterize the nonlocal low-rank property as an additional prior knowledge to decrease the freedom degree of BCS and take K-SVD [24] to estimate the sparisty basis. Refs. [25,26] focused on learning a global block-based analysis operator/dictionary from compressive measurements. However, each block is considered independently in their procedure of reconstruction which ignores the relationship among blocks, resulting in inaccurate sparse coefficients. In contrast, our approach takes advantage of nonlocal low-rank property which can exploit high-order dependency among sparse coefficients of similar blocks as an additional prior knowledge. Moreover, compared with [25,26] which divided the image into many non-overlapping blocks, BCS-BSNLR divides the image into overlapping blocks and groups a set of similar blocks, reducing artifacts in reconstruction image. Since our proposed BCS-BSNLR approach exploits not only block sparsity prior but also nonlocal low-rank prior, it can reduce the required sampling rate and obtain better results.

The reminder of this paper is organized as follows. The proposed BCS based on block sparsity and nonlocal low-rank priors is presented in details in Section 2. In Section 3, we show that the optimization problem can be solved efficiently by the alternating direction multiplier method. In Section 4, we give numerical results demonstrating the effectiveness of the BCS-BSNLR approach. Finally we provide some concluding remarks in Section 5.

2. Problem statement

Assume an image $x \in \mathbb{R}^n$ is of size n, and its sensing measurement matrix is denoted by $\Phi \in \mathbb{R}^{m\times n} (m \ll n)$, BCS aims to recover the image x from the few measurements $y \in \mathbb{R}^m$, i.e. $y = \Phi x$. Since $m \ll n$, the matrix Φ is rank-deficient, there exists more than one $x \in \mathbb{R}^n$ that can yield the same measurements y. To recover original image x from the measurements y, some prior knowledges of x are needed. In this paper, we exploit both block sparsity and low-rank property of the coefficient matrix of each block group.

Suppose $x_j \in \mathbb{R}^d$ denotes an image block of size $\sqrt{d} \times \sqrt{d}$ at position $j\,(j = 1, 2 \ldots e)$, where d is the size of each block vector and e is the number of image blocks. For each block $x_j \in \mathbb{R}^d$, it searches in its neighborhood for h best matched blocks such that each matched block x_{j_c} satisfies $\|x_j - x_{j_c}\|_2 < \varepsilon_0$, where ε_0 is a predefined threshold. These matched blocks form the j-th block group $X_j = [x_{j_1}, x_{j_2}, \ldots x_{j_h}] \in \mathbb{R}^{d\times h}$, which has a low-rank property. Then the recovery problem is formulated:

$$(x, \Lambda_j, D_j) = \arg\min_{x,\Lambda_j,D_j} \|y - \Phi x\|_2^2 + \sum_j (\lambda rank(\Lambda_j) + \kappa\|\Lambda_j\|_0) \quad s.t. \;\; P_j x = D_j \Lambda_j \tag{2}$$

where $P_j = [p_{j_1}, p_{j_2}, \ldots p_{j_h}]$ is an operator that exacts the block group X_j from the image x and p_{j_c} is the block exaction operator. D_j is an unknown sparsity basis in which each block group X_j can be sparsely represented and Λ_j is the sparse vector that satisfies $X_j = D_j\Lambda_j$. $rank(\cdot)$ is used to calculate the rank of the matrix. In the next Section, we will show that the proposed objective function can be efficiently solved by the method of alternative minimization. Fig. 1 gives the flow chart of our BCS-BSNLR.

3. Optimization algorithm

It is very difficult to solve the above constrained optimization problem consisting of rank regularization terms. So we employ the alternating direction method of multipliers (ADMM), which has been widely used in compressive sensing [15], to divide this complicated problem into simpler sub-problems and address them iteratively. By adding a set of auxiliary variables $\{Z_j\}$, the recovery problem can be reformulated as

$$(x, \Lambda_j, Z_j, D_j) = \arg\min_{x,\Lambda_j,D_j} \|y - \Phi x\|_2^2 + \sum_j (\lambda rank(Z_j) + \kappa\|\Lambda_j\|_0) \quad s.t. \;\; P_j x = D_j\Lambda_j, \Lambda_j = Z_j \tag{3}$$

This objective function given has the desirable property that is separable in four variables. Thus, this function can be minimized over one variable by fixing the other variables. Let $\{f_j, g_j\}$ be a set of Lagrangian multipliers, we can write the augmented Lagragian function of this constrained problem as follows:

$$(x, \Lambda_j, Z_j, D_j) = \arg\min_{x,\Lambda_j,Z_j,D_j} \|y - \Phi x\|_2^2 + \sum_j \left(\lambda rank(Z_j) + \beta \left\| P_j x - D_j\Lambda_j + \frac{f_j}{\beta} \right\|_F^2 + \kappa\|\Lambda_j\|_0 + \gamma \left\| \Lambda_j - Z_j + \frac{g_j}{\gamma} \right\|_F^2 \right) \tag{4}$$

The optimization of Eq. (4) consists of the following four sub-problems:

$$D_j^{k+1} = \arg\min_{D_j} \left\| P_j x^k - D_j \Lambda_j^k + \frac{f_j^k}{\beta} \right\|_F^2 \tag{5}$$

$$Z_j^{k+1} = \arg\min_{Z_j} \lambda rank(Z_j) + \gamma \left\| \Lambda_j^k - Z_j + \frac{g_j^k}{\gamma} \right\|_F^2 \tag{6}$$

$$\Lambda_j^{k+1} = \arg\min_{\Lambda_j} \kappa\|\Lambda_j\|_0 + \beta \left\| P_j x^k - D_j^{k+1}\Lambda_j + \frac{f_j^k}{\beta} \right\|_F^2 + \gamma \left\| \Lambda_j - Z_j^{k+1} + \frac{g_j^k}{\gamma} \right\|_F^2 \tag{7}$$

$$x^{k+1} = \arg\min_x \|y - \Phi x\|_2^2 + \beta \sum_j \left\| P_j x - D_j^{k+1}\Lambda_j^{k+1} + \frac{f_j^k}{\beta} \right\|_F^2 \tag{8}$$