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Fast computation of the compressive hyperspectral imaging by using alternating least squares methods

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ABSTRACT

Hyperspectral imaging acquires up to several hundreds of narrow and adjacent spectral band images simultaneously. However, since the dimension of the hyperspectral imaging data, which typically forms a third order tensor, is increased in proportion to the size of spatial and the spectral information at the same time, the higher order singular value decomposition (HOSVD) is appropriate to reduce its dimension. One of the simplest and most accurate approaches for computing the HOSVD is higher order orthogonal iteration (HOOI), which computes the factor matrices from the unfolding matrices of the given tensor by using singular value decomposition alternatively until convergence is achieved. However, because of its expensive computational complexity, we propose a faster algorithm to compute the HOSVD even though the output shows no meaningful difference from that obtained by HOOI. Specifically instead of computing the factor matrix from the updated tensor in every iteration along each mode, we reuse the intermediate result after updating one factor matrix to modify the others in a single iteration. Numerical experiments reveal that the proposed algorithm computes the dimension-reduced hyperspectral imaging much faster than HOOI with fewer outer iterations. Moreover, the difference in accuracy between the proposed algorithm and HOOI is negligible.

1. Introduction

Hyperspectral imaging (HSI), also called imaging spectrometer technique, collects and processes information of a target scene across the electromagnetic spectrum. It has been used for various applications, for instance, in face detection, biology, and automatic visual inspection systems, to improve target detection and classification [1], to identify materials [2], or to detect anomalies [3]. The HSI sensor achieves this by acquiring up to several hundreds of narrow and adjacent spectral band images ranging from ultraviolet to far-infrared wavelengths, simultaneously. The HSI disperses the incoming light spectrum into different wavelengths by employing dispersing element such as a prism or grating. This divided light is captured by the CCD or CMOS sensor as a two-dimensional image $\mathbb{R}^{X\times\lambda}$, where X represents the spatial information of the image, and λ indicates the spectral information. Covering the entire area of the target image requires the sensor to capture a snapshot of every line across the target area along the Y-direction, after which it reconstructs the results as a three-dimensional structure $X\times Y\times\lambda$. Because its three-dimensional structure comprises the spatial information $X\times Y$ and spectral ranges λ, multi-way arrays or tensors are expected to be the most appropriate structure to manipulate the HSI data. However, one problem may arise from the fact that large volumes of imaging data and the number of spectral bands require considerable computational resources while manipulating a tensor, thus some compression techniques that are computationally efficient must be considered. Specifically, we consider Tucker decomposition to reduce the dimension of the given HSI which enables filtering out the unnecessary part of the tensor such as noise.

Together with the CANDECOMP/PARAFAC decomposition for tensor analysis, Tucker decomposition is probably one of the most widely used tensor decomposition techniques due to the fact it was originally invented for application to psychometrics by Tucker [4,5]. Since then it has found numerous applications, such as signal processing [6,7], image processing and computer vision [8,9] as well as dimension reduction in HSI, owing to its simplicity and ease of use. Tucker decomposition is used to decompose a higher-order tensor into a core tensor multiplied by a matrix along each mode. For example, consider an Nth order tensor $\mathcal{T}\in\mathbb{R}^{I_1\times\cdots\times I_N}$, where $I_n, 1\le n\le N$, represents the size of the nth dimension at $\mathcal{T}$. Tucker decomposition decomposes $\mathcal{T}$ as

$$\mathcal{T}=\mathcal{G}\cdot_{n=1\text{ to }N}\mathbf{U}_n, \tag{1}$$

where $\mathbf{U}_n\in\mathbb{R}^{I_n\times R_n}, 1\le n\le N$ represents the factor matrix, and the tensor $\mathcal{G}\in\mathbb{R}^{R_1\times\cdots\times R_N}$ denotes the core tensor, respectively. If the

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multilinear ranks $(R_1, \ldots, R_N)$ are less than $(I_1, \ldots, I_N)$ respectively, then the core tensor $\mathcal{G}$ is considered as the compressed tensor of $\mathcal{T}$.

As the constraint Tucker decomposition, the higher order singular value decomposition (HOSVD) restricts its factor matrices to be orthogonal. Specifically, the factor matrix $\mathbf{U}_n, 1 \leq n \leq N$ in (1) satisfies $\mathbf{U}_n^T\mathbf{U}_n = I$, and the core tensor $\mathcal{G}$ has the property of ordering and all-orthogonality: any two slices of $\mathcal{G}$ are orthogonal [10]. Since a general Tucker decomposition can be converted into an orthogonal Tucker decomposition with an equal decomposition error, we consider the HOSVD to compute factor matrices and a core tensor to compute the dimension reduced HSI tensor. The simplest way of computing the HOSVD is to compute the leading singular vectors of each unfolding matrix from $\mathcal{T}$ [10], however, it does not guarantee that the solution is optimal. Later, De Lathauwer et al. proposed a more accurate technique for calculating factor matrices and a core tensor with an orthogonal constraint, which was denoted as the higher order orthogonal iteration (HOOI) [11]. This algorithm has a limitation as well, since it requires the iterative computation of the singular value decomposition (SVD) from the large matrices unfolded from the tensor. Therefore, it demands substantial amount of computational resources. HOOI has another limitation, because its output may not be optimal. However, since HOOI produces more accurate results than HOSVD as mentioned in [10] in most cases, it is still one of the most well-known methods to compute the HOSVD.

In this letter, our method reduces the spectral and spatial dimensions of the HSI data in a fast and efficient manner. Specifically, we achieve faster computation than HOOI without using SVD, and apply it to the dimension reduction of the HSI data. Consequently, it can be used to improve the performance of denoising HSI, preprocess before the object detection in HSI, or transmit large volumes of imaging data. The remainder of this letter is organized as follows: Section 2 presents some definitions and explanations of basic tensor algebra. Section 3 briefly introduces the HOOI algorithm. In Section 4, we propose the algorithm that enhances the computational speed to compute the HOSVD. Section 5 contains numerical experiments to enable a comparison of the performance with that of the other HOSVD algorithms. In Section 6, we conclude this study.

2. Notations and preliminaries

We start by defining the symbols and terminologies that are consistently used in the letter. We ensure that components of Tensor Algebra are distinguished more easily by denoting these tensors by calligraphic letters, e.g., $\mathcal{T}$. Matrices, vectors, and scalars are written in boldface capital letters (e.g. $\mathbf{T}$), boldface lowercase letters (e.g. $\mathbf{t}$), and lowercase letters (e.g. t), respectively.

The mode-n product of the tensor $\mathcal{T}$ by the matrix $\mathbf{A} \in \mathbb{R}^{R_n \times I_n}$ generates the tensor in $\mathbb{R}^{I_1 \times \cdots \times I_{n-1} \times R_n \times I_{n+1} \times \cdots \times I_N}$ with updated elements such that

$$(\mathcal{T} \cdot_n \mathbf{A})_{i_1 \ldots i_{n-1} r_n i_{n+1} \ldots i_N} = \sum_{i_n=1}^{I_n} x_{i_1 i_2 \ldots i_N} a_{r_n i_n},$$

for all $r_n, 1 \leq r_n \leq R_n$.

The Frobenius norm of the tensor $\mathcal{T} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is the square root of the squares of all elements in $\mathcal{T}$ such that

$$\|\mathcal{T}\|_F = \sqrt{\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \cdots \sum_{i_N=1}^{I_N} x_{i_1 i_2 \ldots i_N}^2}.$$

The tensor $\mathcal{T}$ can be flattened as the matrix form,

$$\mathbf{T}_n \approx \mathbf{U}_n \mathbf{G}_n (\otimes_{i=1 \text{ to } N, i \neq n} \mathbf{U}_i^T), \tag{2}$$

where $\mathbf{T}_n$ indicates the mode-n unfolding matrix, which reorders its elements of $\mathcal{T}$ into the matrix $\mathbf{T}_n$ properly [10]. Likewise, $\mathbf{G}_n$ represents the mode-n unfolding matrix of $\mathcal{G}$. We note that the symbol $\otimes$ denotes the Kronecker product. Assume we have invertible matrices $\mathbf{A}$ and $\mathbf{B}$. The Kronecker product has the following useful properties.

$$\begin{aligned} (\mathbf{A} \otimes \mathbf{B})^T &= \mathbf{A}^T \otimes \mathbf{B}^T, \\ (\mathbf{A} \otimes \mathbf{B})^{-1} &= \mathbf{A}^{-1} \otimes \mathbf{B}^{-1}. \end{aligned} \tag{3}$$

3. Higher order orthogonal iteration

In this section, we briefly summarize the HOOI algorithm. Let the orthogonal factor matrices of $\mathcal{T}$ be $\mathbf{U}_n, 1 \leq n \leq N$. Then the formulation of the optimization problem of finding $\mathbf{U}_n$ and the core tensor $\mathcal{G}$ is given by

$$\begin{aligned} &\min_{\mathcal{G}, \mathbf{U}_n, n=1 \text{ to } N} \|\mathcal{T} - \mathcal{G} \cdot_{n, n=1 \text{ to } N} \mathbf{U}_n\|_F, \\ &\text{subject to } \mathbf{U}_n^T \mathbf{U}_n = I. \end{aligned} \tag{4}$$

Since $\mathcal{G}$ satisfies $\mathcal{G} = \mathcal{T} \cdot_{n, n=1 \text{ to } N} \mathbf{U}_n^T$ from (2) and (3), we reformulate the optimization problem (4) as

$$\|\mathcal{T} - \mathcal{G} \cdot_{n, n=1 \text{ to } N} \mathbf{U}_n\|_F = \|\mathcal{T}\|_F - \|\mathcal{G}\|_F,$$

and the equation (4) is equivalent to finding the solution of maximizing the problem

$$\max \|\mathcal{T} \cdot_{n, n=1 \text{ to } N} \mathbf{U}_n^T\|_F. \tag{5}$$

If we matricize (5) with mode-n such that

$$\max \|\mathbf{U}_n^T \mathbf{S}\|_F,$$

where $\mathbf{S} = \mathbf{T}_n \otimes_{i=N \text{to} 1, i \neq n} \mathbf{U}_i$, then $\mathbf{U}_n$ is obtained from the singular value decomposition (SVD). By fixing the other factor matrices, we can compute $\mathbf{U}_n$ from the R_n leading singular vectors of $\mathbf{S}$ while satisfying the orthogonal constraint. Once $\mathbf{U}_n$ is computed, the next step is to find the other factor matrix along each mode while fixing $\mathbf{U}_n$. This procedure is continued until it converges to the solution.

The HOOI algorithm enables the computation of factor matrices and a core tensor from a tensor in the sense of simplicity and efficiency. Subsequently, many algorithms have been proposed to speed up the HOOI algorithm. For example, Eldén and Savas introduced the Newton–Grassmann optimization, which requires less iterations than HOOI and converges numerically to the solution with quadratic convergence [12]. Ishteva, et al. proposed the Riemannian trust-region-based algorithm with the similar aim of reducing the number of iterative cycles [13]. However, those methods demand very expensive computations in a single iterative step. Phan et al. suggested a fast algorithm of HOSVD by using a Crank–Nicholson-like algorithm in order to avoid using SVDs [14].

4. Derivation of the algorithm

The proposed algorithm was motivated by the idea that the result of updating one factor matrix affects other factor matrices such that all factor matrices can be modified in a single iterative step. This can be expected to lead to faster convergence to the solution with less outer iteration than is required by HOOI. Additionally, we pursue using simple computation of the factor matrices rather than using the SVD so that most of the computations exploit matrix–matrix multiplications instead of matrix decomposition. For simplicity, and to apply the algorithm to the dimension reduction of HSI , we consider the tensor $\mathcal{T}$ as a third-order tensor with the size $I_1 \times I_2 \times I_3$. Rewrite the optimization problem of finding the orthogonal factor matrices in (4) as

$$\min_{\mathcal{G}, \mathbf{U}_n, n=1 \text{ to } 3} \|\mathcal{T} - \mathcal{G} \cdot_{n=1 \text{ to } 3} \mathbf{U}_n\|_F. \tag{6}$$

We next explain the computation of the factor matrices $\mathbf{U}_n$ from the order of (1,2,3) mode in a single iterative step as an example. Without loss of generality, the order can be interchangeable in different ways.