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A fusion estimation method based on fractional Fourier transform

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ABSTRACT

Image denoising methods have different denoising performance in both spatial and transform domains, and each method has its relative advantages and inherent shortcomings compared with other methods. A very intuitive idea is to find that an effective fusion method that can combine with the advantages of different denoising methods. In this paper, we propose a novel fusion method based on the fractional Fourier transform and apply it to image denoising problem. Our method is mainly divided into three steps: Firstly, a pre-estimation is made by any two denoising method separately in the spatial domain. Secondly, using these two estimated results as well as their Fourier transform, twice Fourier transform and three times Fourier transform, we obtain a fused result in the fractional Fourier transform domain. Thirdly, the inverse fractional Fourier transform and the modulus operation are used to obtain the final fusion result. Obviously, this approach is the fusion method in four different domains. Experimental results on benchmark test images demonstrate that the proposed method outperforms state-of-the-art stand-alone methods as: BM3D, DDID, MLP, EPLL and also superior to the fusion methods such as classic wavelet fusion method, PCA fusion method and the state-of-the-art CIEM fusion method in terms of quantity value such as the peak signal to noise ratio (PSNR), the structural similarity (SSIM), and visual quality.

1. Introduction

Generally speaking there are two main classes of image denoising methods: the methods in the spatial domain and the methods in the transform domain. The spatial domain methods include Total Variation based methods [1–4], Non-Local based methods [5–8], and other variational methods. The transform domain methods work by compressing/thresholding coefficients in some transform domains [9–12], which can preserve details like textures, but suffer from artifacts near edges like ringing effects. In contrast, the spatial domain methods often preserve features like edges, but they have difficulties in preserving low contrast details. For example, Total Variation minimization method preserves features like edges, but it has serious staircase effects.

In order to solve these problems, many hybrid methods are taken by recent works. Inspired by non-local means algorithm, Dabov et al. [13,14] proposed a milestone image denoising algorithm named block-matching and 3-D filtering (BM3D). The algorithm is realized by block-matching, collaborative filtering and shrinkage in a 3-D transform domain. It can excellently remove additive noise. Buades et al. [15,16] proposed a multi-layer perceptron (MLP) to automatically learn a denoising method. This method is learned directly with plain multi-layer perceptrons applied to image patches drawn from large image database. This method is only for learned neural network. Knauset et al. [17] presented the dual domain image denoising (DDID) method that is competitive in quality with BM3D, but it is much simpler to implement. They combine two popular filters, the bilateral filter in the spatial domain and the short-time Fourier transform with wavelet shrinkage in the transform domain, to produce a new one which has better results. Zoran et al. [18] proposed a generic framework which allows the use of patch models for whole image restoration using any patch based prior which can be described by a mixture of Gaussians (EPLL). In section 2.1, we will compare these methods, and find that they have different results in the frequency domain, especially in the high frequency parts. Therefore, these differences can be used to enhance the denoising effects if we choose the right fusion method.

The classical fusion methods include multi-scale transform methods and principal component analysis method (PCA) etc. Recently, Burger et al. [19] proposed a rule-based scheme to combine internal and external denoising methods (CIEM). They spent a long time to learn different neural network at different noise levels. This method has a good denoising effect, but it is complicated and only

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applied to certain noise level. In order to solve these problems, we propose a novel fusion estimation method based on fractional Fourier transform. It is competitive in quality with CIEM, but it is much simpler to implement.

The fractional Fourier transform, as a generalization of the classical Fourier transform, appears in the mathematical literature as early as 1929 [20]. In 2001, Ozaktas et al. [21] published a monograph about the investigations of fractional Fourier transform and its applications in digital and optical information processing.

But to the best of our knowledge, the fractional Fourier transform as a fusion tool has yet to appear in image processing. In this paper, we first use it as a fusion estimation method and apply it to image denoising problem. The proposed method is mainly divided into three steps. Firstly, a pre-estimation is made by any two denoising method separately in the spatial domain. Secondly, using these two estimated results as well as their Fourier transform, twice Fourier transform and three times Fourier transform, we obtain a fused result in the fractional Fourier transform domain. Thirdly, the inverse fractional Fourier transform and the modulus operation are used to obtain the final fusion result. Therefore, this scheme is the fusion method in four different domains, which can make full use of information in different domains. In experimental part, we combine BM3D with DDID, BM3D with MLP, and BM3D with EPLL in our method, respectively, and experimental results on benchmark test images demonstrate that our method outperforms state-of-the-art stand-alone methods: BM3D, DDID, MLP, EPLL, and also superiors to the fusion methods such as classic wavelet fusion method [22], PCA fusion methods [23] and the state-of-the-art CIEM fusion method, in terms of the peak signal to noise ratio (PSNR), the structural similarity (SSIM), and visual effects.

The rest of this paper is organized as follows. In section 2, we briefly give the analysis of related methods, and introduce the definition of fractional Fourier transform. In section 3, we describe the commonly several fusion methods and our fractional Fourier fusion method in detail. In section 4, the superiorities of the new algorithm are shown through a large amount of numerical experiments, including a comparison with other denoising methods. In section 5, conclusions are summarized.

2. Related work

2.1. The analysis of BM3D, DDID, MLP and EPLL methods

We denoise an image corrupted by zero mean additive white Gaussian noise, which can be formulated as:

$$u = u_0 + n \tag{1}$$

where u is the observed image, u_0 represents the original image and n is the independent identically distributed additive white Gaussian noise with zero mean and variance σ_n^2. We evaluate denoising performance in terms of the peak signal to noise ratio (PSNR) [24] and the structural similarity (SSIM) [25].

The PSNR and SSIM are defined as

$$PSNR = 20\log_{10}\left(\frac{255}{\sqrt{(1/N \times M)\|x-y\|^2}}\right)$$

$$SSIM = \frac{4\sigma_{xy}\mu_x\mu_y}{(\mu_x^2+\mu_y^2)(\sigma_x^2+\sigma_y^2)}$$

where $x, y, N \times M, \sigma_i$ $(i = x, y)$, and μ_i $(i = x, y)$ are the original image, the denoised image, the image size, the standard deviation and the mean of the image, respectively, σ_{xy} is the covariance of x and y.

As well known, BM3D, DDID, MLP and EPLL denosing methods are based on different principle, which leads to different denoising

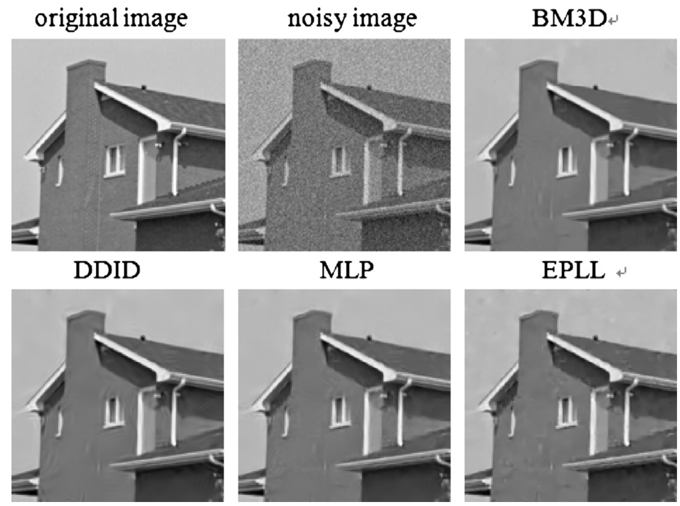

Fig. 1. The denoised images of BM3D, DDID, MLP and EPLL methods for noisy image 'house' with $\sigma = 25$.

Table 1
PSNR (dB) and SSIM of different denoising methods.

	BM3D	DDID	MLP	EPLL
PSNR	32.86	32.66	32.57	32.04
SSIM	0.8589	0.8530	0.8545	0.8469

performance in the spatial domain and in the transform domain. For example, we analysis the denosing results of these methods on image 'house' with noise level $\sigma_n = 25$ in Fig. 1. Table 1 lists the denoised results of different methods in terms of PSNR and SSIM in the spatial domain. Fig. 2 shows the spectrums of the denoised results in the transform domain. The center part is the low frequency of image, and the surrounding part is the high frequency. From the Fig. 1, it is difficult to find the differences among the various denoised results. But these differences can be obvious seen in Table 1 and Fig. 2. It can be found that the denoised result of BM3D method is the best in terms of PSNR and SSIM, but the difference of its spectrum is the biggest compared with the spectrum of the original image. On the contrary, the denoised result of EPLL method is the worst in terms of PSNR and SSIM, but its spectrum is the most similar to the spectrum of the original image. Fig. 2 shows the main differences are in high frequency parts. In order to further explain the difference of the high frequency parts, we first extract the high frequency by employing the high-pass filter of radius 50, then choose the upper left blocks of size 50 × 50 in each high frequency parts, and finally calculate the ratios of the energy of these blocks accounting for the total energy of corresponding high frequency parts, which are 0.0437, 0.0431, 0.0428, 0.0435, 0.0429, 0.0436, respectively.

From what has been discussed above, if we choose the proper fusion method to make full use of these differences in the frequency domain and in the spatial domain, the denoising effects may be further improved. Fortunately, the fractional Fourier transform can fully combine the frequency information with the spatial information. For this purpose, we first introduce the fractional Fourier transform.

2.2. Definition of fractional Fourier transform

As we all known, Fourier analysis is one of the most frequently used tools in signal processing and many other scientific fields. Two functions $f \in L^2(R)$ and F are the Fourier transform pair if they satisfy