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Uniform exponential stability of periodic discrete switched linear system

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Abstract

Let $\{\Pi_\tau(m, n)\colon m \geq n \geq 0\}$ be the family of periodic discrete transition matrices generated by bounded valued square matrices $\Lambda_{\tau(n)}$, where $\tau : [0, 1, 2, \dots) \to \Omega$ is an arbitrary switching signal. We prove that the family $\{\Pi_\tau(m, n)\colon m \geq n \geq 0\}$ of bounded linear operator is uniformly exponentially stable if and only if the sequence $n \mapsto \sum_{k=0}^{n} e^{i\alpha k}\Pi_\tau(n, k)w(k) : \mathbb{Z}_+ \to \mathbb{R}$ is bounded.

1. Introduction

A switched system is a dynamical system that consists of several continuous-time or discrete-time subsystems and a logical rule, called a switching signal, that determines the switching manner among the subsystems [32,34,39]. The switched systems have numerous applications in the control of mechanical systems, chemical processes, automotive industry, power systems, aircraft and traffic control, networked control system and many other fields [11,44,45]. Switched systems with all subsystems described by linear differential or difference equations are called switched linear systems [4–6,15,18].

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In recent years, research work specially focus on the analysis of dynamic behaviors of switched linear system, such as stability [9,15,18,21,23], controllability, reachability [7,16,17,31,33], and observability [5,10,14], etc. Stability analysis is a fundamental issue among numerous interesting topics on switched systems. Stability of switched systems has been classified into two classes, stability under arbitrary switching, which is concerned with finding conditions that guarantee asymptotic stability of a switched system under all possible switching signals [21, Chapter 2], and stability under constrained switching, which is concerned with identifying classes of switching signals for which the switched system is asymptotically stable [21, Chapter 3].

For the stability analysis problem under arbitrary switching, it is necessary to require that all the subsystems are asymptotically stable. However, even when all the subsystems of a switched system are exponentially stable, it is still possible to construct a divergent trajectory from any initial state for such a switched system. Therefore, in general, the above subsystems stability assumption is not sufficient to assure stability for the switched systems under arbitrary switching, except for some special cases [25,40–42]. Stability analysis under constrained switching has focussed on the concept of slow switching among asymptotically stable systems. A number of papers published on finding appropriate switching strategy in order to stabilize the system, e.g., [26,27,36]. To characterize systems which are asymptotically stable for any arbitrary switching signal, e.g., [13,24,25,29,30]. In [1] and [22] Lie-algebraic conditions are given which yields the existence of a common quadratic Lyapunov-function. Dayawansa and Martin [8] proved that the uniform asymptotic stability implies the existence of a common Lyapunov function for compact linear poly-systems. Moreover, other different methods are developed for the stability analysis of switched system by different mathematician like average dwell time method and mode-dependent average dwell time method (see [19,20,37,38,43]).

On the other hand, the stability of periodic switched system was investigated by different researchers with different approaches, see e.g., [2,3,12,28,35]. This type of systems can be used to model switching amplifier, switch mode power supplier, sampled data controlled system and switch mode capacitors filter. To this end, our goal is to state sufficient conditions under which the switched system is stable. We introduce a more general stability criterion for the periodic discrete switched system in term of the boundedness of solutions of non-homogeneous periodic switched linear system with initial condition.

2. Notation and preliminaries

Throughout this paper, we use $\mathbb{Z}_+$, $\mathbb{R}$ and Δ to denote the set of positive integers, the set of real numbers and the set of $n \times n$ matrices $\{\Lambda_\kappa\}_{\kappa \in \Omega}$, respectively, where Ω is an arbitrary set of indices, $\|\cdot\|$ denotes the Euclidean norm. We define a piece wise constant map $\tau : [0, +\infty] \to \Omega = \{1, 2, 3, \ldots, N\}$ known as switching signals.

The solution of the discrete switched linear systems $\Psi(n+1) = \Lambda_{\tau(n)}\Psi(n)$ or $\Psi(n+1) = \Lambda_{\tau(n)}\Psi(n) + e^{i\alpha(n+1)}w(n+1)$, where α denotes any real number, leads to the idea of discrete transition matrix. The family $\prod := \{\Pi_\tau(n, m) : n, m \in \mathbb{Z}_+, n \geq m\}$ of bounded linear operators is called family of $\mathcal{M}$-periodic discrete transition matrix, for a fixed integer $\mathcal{M} \in \{2, 3, \ldots\}$, if it satisfies the following properties:

- $\Pi_\tau(n, n) = I,$ for all $n \in \mathbb{Z}_+$.
- $\Pi_\tau(n, r)\Pi_\tau(r, m) = \Pi_\tau(n, m),$ for all $n \geq r \geq m,\ n, m, r \in \mathbb{Z}_+$.
- $\Pi_\tau(n + \mathcal{M}, m + \mathcal{M}) = \Pi_\tau(n, m),$ for all $n \geq m,\ n, m \in \mathbb{Z}_+$.

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