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An alternative representation of the receptance: The 'elliptical plane' and its modal properties

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ABSTRACT

Modal Identification from Frequency Response Functions (FRFs) has been extensively investigated up to the point its research reached a stagnation state. Yet, a new approach to determine the modal damping factors from FRFs was recently proposed, showing that there still is scope for new findings in the field. Contrary to other modal identification methods which are based on the dynamic motion governing equations, the method used the dissipated energy per cycle of vibration as a starting point. For lightly damped systems with conveniently spaced modes, it produced quite accurate results, especially when compared to the well-known method of the inverse. The method used a plot of the sine of the phase of the receptance against its amplitude, whereby damping was determined from the slope of a linear fit to the resulting plot. In this paper, it is shown that this plot has other (perhaps more important) special properties that were not explored before. Near resonant frequencies, its shape is elliptical, whereby the real and imaginary parts of the modal constants can be determined from numerical curve-fitting. This finding allowed developing a new method which formulation is presented in this paper. The method is discussed through numerical and experimental examples. Although the intention is not to present a new modal identification method that is superior to other existing ones (like the method of the inverse or those based on the Nyquist plot), the authors believe that this new representation of the receptance and its properties may bring valuable insights for other researchers in the field.

1. Introduction

The interest of modal identification procedures is acknowledged by the scientific community and many authors have addressed this problem, mainly since the early seventies of the past century [1]. The existing to date modal identification procedures cover different levels of sophistication and, in almost all cases, require the use of software that may not be easy to obtain.

In the past few years, attention has been more focused on Operational Modal Analysis (OMA) rather than in the more traditional Experimental Modal Analysis (EMA). Examples of later developments in OMA identification methods can be found, for instance, in [2–5]. In terms of EMA, later publications are more concerned with Engineering applications or dealing with

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uncertainty in existing methods, as can be seen, for instance, in [6,7]. OMA deals with operational deflection shapes and many often make use of output-only measurements, this meaning that excitation loads are unknown. EMA makes use of both input forces and output responses in order to determine modal parameters and mode shapes. Numerous modal identification algorithms have been developed in the past thirty years [8]. However, even if in the past recent years not many advances have been seen in terms of EMA modal identification methods, there are a few interesting results that can still be derived.

If the sole objective is the determination of the global modal characteristics, it is possible to use simple approaches that produce quick estimates of the desired information. The issue of determining the modal damping factors has recently been presented [9,10] from a different perspective. In that new approach, the starting point was the dissipated energy per cycle of vibration rather than the governing equations of the dynamic motion as it is usually done. The proposed methodology is based on a special plot of the receptance, whereby the vertical axis is the sine of the phase angle and the horizontal axis is the amplitude (in a similar fashion to what is done with the Nyquist plot of the receptance).

As it is shown in this paper, this plot has special properties, one of which is that the data points around a resonant frequency describe a loop that resembles the half of an ellipse. It is also shown that the major and minor axis of the ellipse are related to the modal constants, which can be determined using numerical extrapolation methods. Modal constants are important because they contain information about the mode shapes (local modal characteristics) which, in the general case of non-proportional damping, are complex quantities [1]. Modal identification techniques seek to extract from experimental data the modal parameters that characterise the dynamic behaviour of a structure [11]. Thus, a modal model is derived through the determination of the values (for each mode of vibration) of the natural frequency (not explored in this paper), damping ratio (explored in [9,10]) and complex modal constant (explored in this paper). Once the modal model is known, it can be used in many different fields of structural dynamics, e.g., structural modification, coupling techniques (including mass cancelation), finite element updating, transmissibility and structural health monitoring, just to mention a few [1,12–18].

This paper presents the development of the new proposed methodology for the determination of the modal constants and illustrates its application through both numerical and experimental examples. It is important to note that the intention is not to present a new approach to modal identification that is superior to existing ones (like the method of the inverse or those based on the Nyquist plot). However, the fact that this new representation of the receptance in an 'elliptical plane' is a function of the modal properties, is a reason for the authors to believe that this research may bring valuable insights to other researchers in the field.

2. Theoretical development

The development presented in And 2.12.2 is not original and has been previously presented in detail in [10]. However, the authors considered it would be important to summarise it here so that the whole of the proposed method can be better understood.

2.1. Definitions

Let us start from the well-known second-order differential equation of motion for a single degree of freedom (SDOF) viscous system given by:

$$m\ddot{x} + c\dot{x} + kx = Fe^{i\omega t} \tag{1}$$

where m is the mass, c is the viscous damping coefficient, k is the stiffness, F is the amplitude of the oscillatory force, t is the time variable and $i = \sqrt{-1}$ is the imaginary number. When excited by an harmonic force at a frequency ω, it can be easily proven (and most fundamental texts on vibration theory show it, for instance [1,14]) that for each vibration cycle the system dissipates – through its viscous damper – a quantity of energy directly proportional to the damping coefficient, the excitation frequency and the square of the response amplitude X:

$$W_{diss} = \int_0^T f\dot{x}ht = \pi c\omega X^2 \tag{2}$$

where $T = 2\pi/\omega$ is the time period of oscillation. However, experimental evidence from tests performed on a large variety of materials show that the damping due to internal friction (material hysteresis) is nearly independent of the forcing frequency but still proportional to the square of the response amplitude [19], i. e.:

$$W_{diss} \propto CX^2 \tag{3}$$

where C is a constant. Therefore, from Eqs. (2) and (3) the equivalent damping coefficient is:

$$c = \frac{C}{\pi\omega} = \frac{h}{\omega} \tag{4}$$

where h is the hysteretic damping coefficient. In such conditions, Eq. (1) can be re-written as: