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A subsystem identification method based on the path concept with coupling strength estimation

Francesc Xavier Magrans [a,b], Jordi Poblet-Puig [a,*], Antonio Rodríguez-Ferran [a]

[a] Laboratori de Càlcul Numèric, E.T.S. d'Enginyers de Camins, Canals i Ports de Barcelona, Universitat Politècnica de Catalunya, Jordi Girona 1, Campus Nord, E-08034 Barcelona, Spain

[b] Ingeniería para el Control del Ruido, Berruguete 52, Vila Olímpica (Vall d'Hebron), E-08035 Barcelona, Spain

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ABSTRACT

For complex geometries, the definition of the subsystems is not a straightforward task. We present here a subsystem identification method based on the direct transfer matrix, which represents the first-order paths. The key ingredient is a cluster analysis of the rows of the powers of the transfer matrix. These powers represent high-order paths in the system and are more affected than low-order paths by damping.

Once subsystems are identified, the proposed approach also provides a quantification of the degree of coupling between subsystems. This information is relevant to decide whether a subsystem may be analysed in a computer model or measured in the laboratory independently of the rest or subsystems or not. The two features (subsystem identification and quantification of the degree of coupling) are illustrated by means of numerical examples: plates coupled by means of springs and rooms connected by means of a cavity.

1. Introduction

The subdivision of a mechanical system into subsystems according to their vibroacoustic response is a task required in several modelling methods or in order to better understand the behaviour of the mechanical system. A clear example is Statistical Energy Analysis (SEA, [1,2]) that requires, as a preliminary step, the definition of subsystems which satisfy several physical properties (high modal density, equipartition of energy between modes, equal probability of mode excitation, weak coupling between subsystems [3]). Also from an experimental viewpoint it can be interesting to know which parts of a large system (train coach, building, airplane) can be tested in the laboratory isolated from the other parts and the results will still be meaningful. A proper subsystem identification determines the quality in the estimation of modal parameters in the experimental or operational modal analysis [4], a review of techniques for parameter and system identification from measured data can be found in [5]. Finally, the knowledge about subsystem plays also an important role in the transfer path analysis and can determine which is the more appropriate technique to apply as well as the quality of the solution [6–8].

The splitting of the domain is sometimes performed by hand, mainly based on experience and intuition. However, more systematic methods have also been proposed. Some of them simply choose the subsystems according to the material: a glass

* Corresponding author.
E-mail addresses: fxmagrans@icrsl.com (F.X. Magrans), jordi.poblet@upc.edu (J. Poblet-Puig), antonio.rodriguez-ferran@upc.edu (A. Rodríguez-Ferran).

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pane, an aluminium panel, etc. [9,10], other are based on energy models [11–13], regard the frequency response function of a system [14], or study the shape of some eigenfunctions of the system [15].

In these last two methods [14,15], cluster algorithms are used. Cluster analysis is a general purpose and very powerful tool that groups sets of objects in a big population according to their similitude [16]. The data to be provided is: the group of objects, the parameters that characterise them, the comparison criteria and how to measure their similitude (distance).

In this work we apply cluster analysis [17] in order to automatically define the subsystems in a vibroacoustic problem. The core of the method is based on the transfer matrices and their powers. This is representative of transmission paths inside the system.

Another goal of the proposal is to quantify the coupling strength between subsystems. To do this we define the coupling strength as the error committed if coupling is disregarded and each subsystem is solved isolatedly from the rest of subsystems: the larger the error, the larger the coupling strength. We do not attempt to give a strict definition of "weak coupling", because whether coupling between subsystems may be disregarded or not depends on the type of analysis: a certain relative error may be admissible for an industrial pre-project but not for the final design.

Some brief background concepts and the core idea of the method are explained in Section 2. The theoretical aspects and approach are illustrated by the numerical examples of Section 3. The concluding remarks of Section 4 close the paper.

2. Methodology

Our approach to automatically identify the subsystems is based on a combination of the powers of the transfer matrix $\mathbf{T}$ and a standard cluster analysis. $\mathbf{T}^k$ is a representation of the transmission paths inside the mechanical system. Detailed developments can be found in [18–21] and the main concepts required in this section are overviewed in Section 2.1. The main idea of the identification method is exposed in Section 2.3.1. This is done in a conceptual way that is sustained latter by the numerical examples of Section 3. Section 2.3.2 explains the relation between this path-based method and other more widely used criteria. Finally, an important aspect is addressed in Section 2.4: the estimation of the coupling strength between subsystems.

2.1. Overview of path analysis

The problem statement as well as a brief summary of the transfer matrix concept is exposed here. The main goal is to see that a solution of a linear system can be expressed by means of the superposition of multiple paths, that is, powers of the transfer matrix [18].

Assume that the mechanical system is properly described by the linear system of equations

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{1}$$

where the system matrix $\mathbf{A}$ can be split into diagonal $\mathbf{D}$, strictly lower triangular $\mathbf{L}$ and upper triangular $\mathbf{U}$ matrices

$$\mathbf{A} = \mathbf{D} + \mathbf{L} + \mathbf{U} \tag{2}$$

Following the usual convention, all vectors are by default column vectors (for instance, $\mathbf{x}$ and $\mathbf{b}$).

The linear system in (1) can be then rewritten as

$$\mathbf{x} = \mathbf{D}^{-1}\mathbf{b} + \mathbf{T}\mathbf{x} \tag{3}$$

where

$$\mathbf{T} = -\mathbf{D}^{-1}(\mathbf{L} + \mathbf{U}) \tag{4}$$

is the transfer matrix.

From Eq. (3), the solution of system (1) may be expressed as

$$\mathbf{x} = (\mathbf{I} - \mathbf{T})^{-1}\mathbf{D}^{-1}\mathbf{b} \tag{5}$$

It has been proved in [18] that the inverse of the matrix $(\mathbf{I} - \mathbf{T})$ can be expressed as the limit of a matrix series as

$$(\mathbf{I} - \mathbf{T})^{-1} = \lim_{m\to\infty}\left(\sum_{k=0}^{m-n}\mathbf{T}^k + \sum_{k=m-n+1}^{m}\gamma_{m-k}\mathbf{T}^k\right) \tag{6}$$

The series in Eq. (6) is unconditionally convergent, without any constraint on the value of $||\mathbf{T}||$, if coefficients γ_{m-k} are properly chosen. For $||\mathbf{T}|| \geqslant 1$, Eq. (6) with the optimal choice of γ_{m-k} is convergent, whereas the standard Neumann series with $\gamma_{m-k} = 1$ diverges. For $||\mathbf{T}|| < 1$, on the other hand, both series are convergent, but the convergence is faster with the optimal γ_{m-k}. These aspects are discussed in detail in [18].

Eq. (6) as well as expressions for the coefficients γ_{m-k} are the main results in [18]. They allow the generalisation of the path superposition idea to any linear mechanical system (and not only to those with convergent Neumann series). So, a the-