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Short communication

Brownian motion with adaptive drift for remaining useful life prediction: Revisited

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ABSTRACT

Linear Brownian motion with constant drift is widely used in remaining useful life predictions because its first hitting time follows the inverse Gaussian distribution. State space modelling of linear Brownian motion was proposed to make the drift coefficient adaptive and incorporate on-line measurements into the first hitting time distribution. Here, the drift coefficient followed the Gaussian distribution, and it was iteratively estimated by using Kalman filtering once a new measurement was available. Then, to model nonlinear degradation, linear Brownian motion with adaptive drift was extended to nonlinear Brownian motion with adaptive drift. However, in previous studies, an underlying assumption used in the state space modelling was that in the update phase of Kalman filtering, the predicted drift coefficient at the current time exactly equalled the posterior drift coefficient estimated at the previous time, which caused a contradiction with the predicted drift coefficient evolution driven by an additive Gaussian process noise. In this paper, to alleviate such an underlying assumption, a new state space model is constructed. As a result, in the update phase of Kalman filtering, the predicted drift coefficient at the current time evolves from the posterior drift coefficient at the previous time. Moreover, the optimal Kalman filtering gain for iteratively estimating the posterior drift coefficient at any time is mathematically derived. A discussion that theoretically explains the main reasons why the constructed state space model can result in high remaining useful life prediction accuracies is provided. Finally, the proposed state space model and its associated Kalman filtering gain are applied to battery prognostics.

1. Introduction

In the research community of prognostics and health management [1], the remaining useful life (RUL) prediction [2–4] under linear Brownian motion with constant drift [5,6] has attracted much attention because its first hitting time follows the inverse Gaussian distribution given a soft failure threshold. Here, the first hitting time is defined as the time when linear Brownian motion hits the soft failure threshold for the first time. Therefore, the difference between the first hitting time and the current prediction time can be regarded as the RUL. Nevertheless, linear Brownian motion with constant drift cannot be used to model the degradation of a specific product because the drift coefficient established by a population of historical degradation data is always fixed in linear Brownian motion for degradation modelling and RUL prediction. In the pioneering work of Wang et al. [7], the authors constructed a state space model of linear Brownian motion and used Kalman filtering to

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posteriorly estimate the drift coefficient over time when a new measurement was available. As a result, the drift coefficient of linear Brownian motion became adaptive and measurements were incorporated into the first hitting time distribution, such that RUL prediction became more accurate than the idea of linear Brownian motion with constant drift. Following this work, Si et al. [8] extended linear Brownian motion with adaptive drift to nonlinear Brownian motion with adaptive drift so as to model nonlinear degradation. Most importantly, the authors mathematically derived the first hitting time distribution of nonlinear Brownian motion with adaptive drift, which was a milestone in the modelling of nonlinear degradation by using Brownian motion. Since then, many nonlinear Brownian motion-based prognostic methods [9–22] have been proposed and designed to predict the RUL of various products and systems. According to our literature review, the same state space model as that proposed in Ref. [7] was commonly adopted in [9–22] to make these Brownian motion-based prognostic methods adaptive. A flowchart of Brownian motion with adaptive drift-based prognostic methods is plotted in Fig. 1.

The commonly used state space model [7,9–22] is mathematically formulated as follows:

$$\begin{aligned} \lambda_{t_i} &= \lambda_{t_{i-1}} + \eta_{t_i} \\ x_{t_i} &= x_{t_{i-1}} + \lambda_{t_{i-1}} \int_{t_{i-1}}^{t_i} \mu(\tau;\boldsymbol{\theta})d\tau + \sigma_B \varepsilon_{t_i} \end{aligned} \tag{1}$$

where λ_{t_i} is the predicted drift coefficient at the current time t_i before a new measurement x_{t_i} is available; $\lambda_{t_{i-1}}$ is the drift coefficient posteriorly estimated at the previous time t_{i-1} when the previous measurement $x_{t_{i-1}}$ at the previous time t_{i-1} is available; $\mu(\tau;\boldsymbol{\theta})$ is a nonlinear function with parameters $\boldsymbol{\theta}$ used in nonlinear Brownian motion; η_{t_i} is an additive Gaussian process noise drawn from a Gaussian distribution with mean 0 and variance Q; ε_{t_i} is an additive Gaussian observation noise drawn from a Gaussian distribution with mean 0 and variance $(t_i - t_{i-1})$; and σ_B is the diffusion coefficient. If $\mu(\tau;\boldsymbol{\theta})$ is a constant such as 1 without loss of generality, Eq. (1) is exactly reduced to the state space modelling of linear Brownian motion proposed in [7]. Kalman filtering [23] contains two recursive phases, including prediction and update phases, to iteratively estimate the posterior distribution of the drift coefficient λ_{t_i} at the current time t_i when a new measurement x_{t_i} is available.

Prediction phase

The predicted estimate $\widehat{\lambda}_{t_i|t_{i-1}}$ of the drift coefficient λ_{t_i} at time t_i is $\widehat{\lambda}_{t_i|t_{i-1}} = \widehat{\lambda}_{t_{i-1}|t_{i-1}}$ before a new measurement x_{t_i} is available.

The variance $P_{t_i|t_{i-1}}$ of the predicted estimate $\widehat{\lambda}_{t_i|t_{i-1}}$ of the drift coefficient λ_{t_i} at time t_i is $P_{t_i|t_{i-1}} = P_{t_{i-1}|t_{i-1}} + Q$ before a new measurement x_{t_i} is available.

Update phase

The innovation or measurement residual is $\widehat{y}_{t_i} = x_{t_i} - x_{t_{i-1}} - \widehat{\lambda}_{t_{i-1}|t_{i-1}} \int_{t_{i-1}}^{t_i} \mu(\tau;\boldsymbol{\theta})d\tau$.

The posterior estimate $\widehat{\lambda}_{t_i|t_i}$ of the drift coefficient λ_{t_i} at time t_i is $\widehat{\lambda}_{t_i|t_i} = \widehat{\lambda}_{t_i|t_{i-1}} + \overline{K_{t_i}}\widehat{y}_{t_i}$ when a new measurement x_{t_i} is available. The optimal Kalman filtering gain $\overline{K_{t_i}}$ is $\overline{K_{t_i}} = \frac{P_{t_i|t_{i-1}} \int_{t_{i-1}}^{t_i} \mu(\tau;\boldsymbol{\theta})d\tau}{P_{t_i|t_{i-1}} \left(\int_{t_{i-1}}^{t_i} \mu(\tau;\boldsymbol{\theta})d\tau\right)^2 + (\sigma_B)^2 (t_i - t_{i-1})}$.

The variance $P_{t_i|t_i}$ of the posterior estimate $\widehat{\lambda}_{t_i|t_i}$ of the drift coefficient λ_{t_i} at time t_i is $P_{t_i|t_i} = P_{t_i|t_{i-1}} - P_{t_i|t_{i-1}}\overline{K_{t_i}} \int_{t_{i-1}}^{t_i} \mu(\tau;\boldsymbol{\theta})d\tau$.

Solving the state space model provided by Eq. (1) can make some prognostic methods [7,9–22] adaptive, which implies that the drifted coefficient λ_{t_i} at time t_i is posteriorly and iteratively updated by using Kalman filtering and up-to-date measurements $X(t_{0:i}) = \{x_0, x_1, \ldots, x_i\}$. However, an underlying assumption is used in the state space model provided by Eq. (1): in the update phase of Kalman filtering, the predicted drift coefficient λ_{t_i} at the current time t_i exactly equals the posterior drift coefficient $\lambda_{t_{i-1}}$ at the previous time t_{i-1}, which causes a contradiction with the predicted drift evolution driven by the

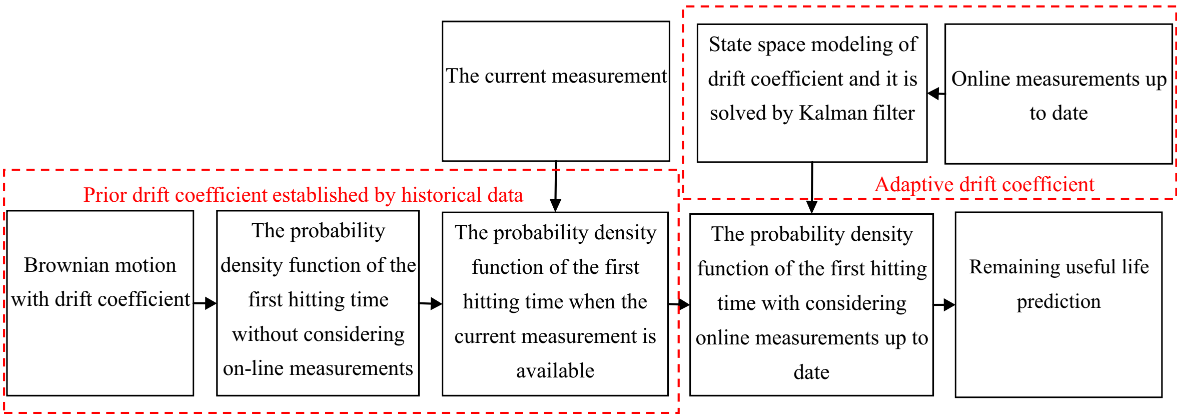

Fig. 1. A flowchart of Brownian motion with adaptive drift-based prognostic methods.