Mechanical Systems and Signal Processing 98 (2018) 548–556

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Vibration fatigue using modal decomposition

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ARTICLE INFO

Article history:
Received 22 July 2016
Received in revised form 14 February 2017
Accepted 31 March 2017
Available online 24 May 2017

Keywords:
Vibration fatigue
Modal model
Spectral moment method
Multiaxial criterion

ABSTRACT

Vibration-fatigue analysis deals with the material fatigue of flexible structures operating close to natural frequencies. Based on the uniaxial stress response, calculated in the frequency domain, the high-cycle fatigue model using the S-N curve material data and the Palmgren-Miner hypothesis of damage accumulation is applied. The multiaxial criterion is used to obtain the equivalent uniaxial stress response followed by the spectral moment approach to the cycle-amplitude probability density estimation. The vibration-fatigue analysis relates the fatigue analysis in the frequency domain to the structural dynamics. However, once the stress response within a node is obtained, the physical model of the structure dictating that response is discarded and does not propagate through the fatigue-analysis procedure. The structural model can be used to evaluate how specific dynamic properties (*e.g.*, damping, modal shapes) affect the damage intensity. A new approach based on modal decomposition is presented in this research that directly links the fatigue-damage intensity with the dynamic properties of the system. It thus offers a valuable insight into how different modes of vibration contribute to the total damage to the material. A numerical study was performed showing good agreement between results obtained using the newly presented approach with those obtained using the classical method, especially with regards to the distribution of damage intensity and critical point location. The presented approach also offers orders of magnitude faster calculation in comparison with the conventional procedure. Furthermore, it can be applied in a straightforward way to strain experimental modal analysis results, taking advantage of experimentally measured strains.

1. Introduction

Structures that vibrate intensely are prone to failure because of material fatigue. Different fatigue-analysis approaches are available to deal with such cases; vibration-fatigue analysis is one that is very suitable when the stress response occurs mainly as a consequence of rich structural dynamics [1,2,30].

In the course of the numerical analysis procedure, the vibration excitation is applied via the frequency-response functions (FRFs) to obtain the distribution of the stress (or strain) tensor in the frequency domain [3,4] for the analyzed structure. Once the stress/strain tensor function of frequency at a particular material point is known, it has to be reduced to an equivalent uniaxial stress function, using a multiaxial criterion [1,5–9], before subsequent vibration-fatigue methods can be applied. Such criteria are actively researched and recent comparative studies [10–14] have shown promising results. One of the simplest and most widely used is the frequency-domain equivalent von Mises (EVMS) criterion proposed by Preumont and

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http://dx.doi.org/10.1016/j.ymssp.2017.03.052
0888-3270/

Nomenclature

${}_rA^s_{ij}$	stress modal constant
${}_r\mathbf{A}^s$	stress modal constants matrix
b	the Tovo-Benasciutti method parameter
C	S-N curve parameter
$\mathbf{C}$	stiffness tensor
d	damage intensity
d_{TB}	damage intensity (Tovo-Benasciutti method)
$d_{\setminus M_r}$	damage intensity for the model excluding mode r
d_{NB}	damage intensity (narrowband approximation)
d_{SO}	damage intensity (Sakai-Okamura method)
D_r	modal damage intensity measure
$\mathbf{D}$	hysteretic damping matrix
$\overline{\mathbf{D}}$	displacements/strain field operator
$\mathbf{f}(t)$	time-dependent force excitation vector
$\mathbf{F}(\omega)$	frequency-dependent force excitation vector
$\mathbf{H}(\omega)$	receptance matrix
$\mathbf{H}_{sf}(\omega)$	stress frequency-response function
$\mathbf{H}^r_{sf}(\omega)$	modal stress frequency-response function
$\widetilde{\mathbf{H}}_{sf}(\omega)$	stress frequency-response function for the reduced model
I^r_i	i-th order, r-th mode int. of ω-dep. part of stress resp.
k	S-N curve slope
$\mathbf{K}$	stiffness matrix
m	number of modes of the reduced modal model
m_i	i-th order spectral moment for the complete model
$\widetilde{m}_i$	i-th order spectral moment for the reduced model
m^r_i	r-th mode i-th order spectral moment
$\mathbf{M}$	mass matrix
N	number of modes of the complete modal model
$\mathbf{Q}$	von Mises criterion constant coefficients matrix
$\widetilde{S}_c(\omega)$	equivalent von Mises multiaxial stress
$S^r_c(\omega)$	equivalent modal von Mises multiaxial stress
$\mathbf{S}_{ff}(\omega)$	forced-excitation power spectral density
$\mathbf{S}_{ff,z}$	forced-excitation power spectral density acting along z-axis
$\mathbf{S}_{ss}(\omega)$	stress-response power spectral density
$\widetilde{\mathbf{S}}_{ss}(\omega)$	stress-response power spectral density of the reduced model
$\mathbf{S}^r_{ss}(\omega)$	modal stress-response power spectral density
t	time
$\mathbf{x}(t)$	time-dependent vector of displacements
$\ddot{\mathbf{x}}(t)$	time-dependent vector of accelerations
$\mathbf{X}$	displacements amplitudes vector
$\mathbf{X}(\omega)$	frequency-dependent displacements amplitude vector
$\mathbf{X}_s(\omega)$	frequency-dependent stress responses
$\widetilde{\mathbf{X}}_s(\omega)$	frequency-dependent stress response of reduced model
α_2	spectral width parameter
$\Gamma(\cdot)$	Euler gamma function
$\zeta_r(\omega)$	ω-dependent part of frequency-response function
η_r	damping loss factor
λ_r	r-th eigenvalue
$\boldsymbol{\phi}_r$	r-th mass normalized eigenvector
$\boldsymbol{\phi}^s_r$	r-th mass normalized stress modal shape
$\mathbf{\Phi}$	modal matrix (of mass-normalized eigenvectors)
$\boldsymbol{\phi}^\varepsilon_r$	r-th mass-normalized strain modal shape
$\mathbf{\Phi}^s$	stress modal matrix
ϕ^s_{ij}	element of stress modal matrix
ϕ_{ij}	element of modal matrix
ψ_r	r-th eigenvector
ω	angular frequency
ω_r	natural frequency