Mechanical Systems and Signal Processing 98 (2018) 570–579

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Full-degrees-of-freedom frequency based substructuring

Armin Drozg, Gregor Čepon *, Miha Boltežar

Faculty of Mechanical Engineering, University of Ljubljana, Slovenia

ARTICLE INFO

Article history:
Received 16 September 2016
Received in revised form 30 March 2017
Accepted 3 April 2017

Keywords:
Piezoelectric rotational accelerometer
Rotational degrees of freedom
Experimental modal analysis
Modal shape slopes
FRF synthesis
Experimental frequency based substructuring

ABSTRACT

Dividing the whole system into multiple subsystems and a separate dynamic analysis is common practice in the field of structural dynamics. The substructuring process improves the computational efficiency and enables an effective realization of the local optimization, modal updating and sensitivity analyses. This paper focuses on frequency-based substructuring methods using experimentally obtained data. An efficient substructuring process has already been demonstrated using numerically obtained frequency-response functions (FRFs). However, the experimental process suffers from several difficulties, among which, many of them are related to the rotational degrees of freedom. Thus, several attempts have been made to measure, expand or combine numerical correction methods in order to obtain a complete response model. The proposed methods have numerous limitations and are not yet generally applicable. Therefore, in this paper an alternative approach based on experimentally obtained data only, is proposed. The force-excited part of the FRF matrix is measured with piezoelectric translational and rotational direct accelerometers. The incomplete moment-excited part of the FRF matrix is expanded, based on the modal model. The proposed procedure is integrated in a Lagrange Multiplier Frequency Based Substructuring method and demonstrated on a simple beam structure, where the connection coordinates are mainly associated with the rotational degrees of freedom.

1. Introduction

The methodology to divide large and complex systems into smaller subsystems is common practice in the field of structural dynamics. Analyzing a subsystem's dynamics separately, results in less complexity and a higher computational efficiency. The dynamic properties can be obtained experimentally or numerically. Whenever the system is modified, the dynamic analysis is reduced only to the given subsystem, which is then merged with the rest of the structure using a dynamic substructuring (DS) process. By using DS methods it is possible to effectively perform a local optimization, modal updating and sensitivity analyses.

Classically, the DS methods can be divided into three subclasses. The first, so-called Component Mode Synthesis (CMS), is based on the modal parameters. The second one, Impulse Based Substructuring (IBS), is the youngest in the family of substructuring methods and is based on the Impulse-Response Functions (IRFs). The last one, Frequency Based Substructuring (FBS), uses the response model within a coupling process. The FBS is normally based on experimental data as there is a possibility to directly measure the Frequency Response Functions (FRFs). Within FBS several methods have been developed; however, until recently, they have not gained much popularity. In 2006, the Lagrange Multiplier Frequency Based

* Corresponding author.
E-mail address: gregor.cepon@fs.uni-lj.si (G. Čepon).

http://dx.doi.org/10.1016/j.ymssp.2017.04.051
0888-3270/

Substructuring (LM FBS) method was introduced by de Klerk et al. [1]. It represents a reformulated version of the admittance FBS method proposed by Jetmundsen et al. [2]. In order to obtain a full-degrees-of-freedom (DOF) FRF matrix, the measurement of translational as well as rotational responses must be performed. This can be done effectively if the FRFs are obtained from a numerical model; however, it still represents a problem whenever the FRFs are obtained solely by experiment. One of the main difficulties is to measure the rotational DOF; thus, it is not possible to obtain the full FRF matrix, which may lead to erroneous results during the substructuring process.

There have been various attempts to excite structures with a pure moment and observe the rotational responses. Some researchers proposed the finite-differences theory, together with precisely positioned translational accelerometers [3,4] to measure the rotational DOFs. Several attempts were made to excite the structure indirectly with the moment by using T-blocks, the finite-differences method or two synchronous impact hammers [5–7]. The development of a rotational sensor based on bimorph materials is presented in [8]. A method to deduce the rotational modal shapes by measuring the strains is shown in [9]. Despite extensive research in this field, the presented methods have limitations and are not yet generally applicable. Thus, some researchers try to combine experimental and numerical approaches [10,11] in order to improve/update the measurements. In some cases the rotations are not even considered in the coupling process [11,12] and only the translational DOFs are used.

The objective of this paper is to present the substructuring process based on an experimentally obtained, full-DOF response model together with the LM FBS method. The rotations are measured using a quartz-based piezoelectric rotational accelerometer that is well established in the car-safety testing industry [13], although it has not yet gained much popularity in the field of experimental modal analysis (EMA). In order to obtain the full-FRF matrix during the substructuring process it is necessary to measure the excitation force as well as the excitation moment. In this paper an alternative approach is proposed where the moment-excited part of the FRF matrix is deduced based on the introduction of the modal model. To generate a complete response model, modal shapes and not generally considered modal-shape slopes will be included in the FRF synthesis algorithm. Moreover, with the curve-fitting process, the procedure makes it possible to smooth the response functions, which additionally improves the quality of the substructuring process.

The following section briefly summarizes the LM FBS method. In Section 3, the inclusion of the rotational degrees of freedom into the response and modal model will be presented. The fourth section presents the characteristics of a piezoelectric rotational accelerometer and, finally, the applicability of the developed method is demonstrated on a simple beam structure.

2. LM FBS method

The LM FBS method requires the full-DOF FRF matrix and the evaluation of the compatibility conditions. The FRFs represent the dynamic stiffnesses between an arbitrary combination of the DOF on the subsystems. With compatibility conditions, one uniquely defines the mutual connections between the subsystems. The theory in this section summarizes the work of de Klerk et al. [1].

The dynamics of an arbitrary subsystem s can be theoretically determined by the equation of motion in the time domain as:

$$\boldsymbol{M}^{(s)}\ddot{\boldsymbol{u}}(t)^{(s)} + \boldsymbol{C}^{(s)}\dot{\boldsymbol{u}}(t)^{(s)} + \boldsymbol{K}^{(s)}\boldsymbol{u}(t)^{(s)} = \boldsymbol{f}(t)^{(s)}, \tag{1}$$

where $\boldsymbol{M}^{(s)}$, $\boldsymbol{C}^{(s)}$ and $\boldsymbol{K}^{(s)}$ represent the mass, damping and stiffness matrices of subsystem s, respectively. The variable $\boldsymbol{f}^{(s)}$ stands for the force excitation vector of the subsystem. Using a Fourier transformation, Eq. (1) can be transformed into the frequency domain as:

$$[-\omega^2\boldsymbol{M}^{(s)} + \mathrm{j}\omega\boldsymbol{C}^{(s)} + \boldsymbol{K}^{(s)}]\boldsymbol{U}(\omega)^{(s)} = \boldsymbol{F}(\omega)^{(s)}, \tag{2}$$

where $\boldsymbol{U}^{(s)}$ generally stands for the displacement, velocity or acceleration vector. Eq. (2) can be further rewritten as:

$$\boldsymbol{Z}(\omega)^{(s)}\boldsymbol{U}(\omega)^{(s)} = \boldsymbol{F}(\omega)^{(s)}, \tag{3}$$

where

$$\boldsymbol{Z}(\omega)^{(s)} = [-\omega^2\boldsymbol{M}^{(s)} + \mathrm{j}\omega\boldsymbol{C}^{(s)} + \boldsymbol{K}^{(s)}], \tag{4}$$

stands for the dynamic stiffness matrix. The mass, damping and stiffness matrices are not separately available when dealing with experimentally obtained data. Thus, Eq. (3) needs to be reformulated in the form:

$$\boldsymbol{Y}(\omega)^{(s)}\boldsymbol{F}(\omega)^{(s)} = \boldsymbol{U}(\omega)^{(s)}, \tag{5}$$

where $\boldsymbol{Y}(\omega)^{(s)}$ is the subsystem's response matrix ($\boldsymbol{Y} = \boldsymbol{Z}^{-1}$). The response matrix contains all the combinations of FRFs between the input excitation and the output response points. These are used in a dual manner formulated LM FBS method.

The coupling process based on the FRF matrix also requires compatibility conditions. The Boolean mapping matrix $\boldsymbol{B}$ defines the subsystems mutual relationship at the connection points. For rigid joint connections, the compatibility matrix is defined as:

$$\boldsymbol{B}\boldsymbol{U} = \boldsymbol{0}. \tag{6}$$