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Spatial-dependent resonance mode and frequency of rotationally periodic structures subjected to standing wave excitation

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ABSTRACT

This work examines the distinct resonance vibration of rotationally periodic structures. An analytical model of a sample stepped-plate structure subjected to standing wave excitation is developed by elasticity theory. Spatial-dependent resonance mode and resonance frequency are formulated by perturbation-superposition method. Different from the natural mode and natural frequency, a sinusoidal fluctuation of the resonance frequency is identified between the two split natural frequencies for single standing wave excitation. The resonance mode does not have preferred orientation because it is determined by excitation orientation. The resonance behaviors are different from those near the repeated natural frequencies. The response to a standing wave pair is also calculated and compared with that to the mathematically equivalent traveling wave, where significant difference is identified. The results indicate that purer traveling wave can be created by using a standing wave pair with pre-selected spatial phase and excitation frequency. Reverse traveling direction can be realized by altering excitation frequency. A test rig is designed and fabricated for verification purpose. The experiment validates that the response near the split natural frequencies is in phase with the external standing wave excitation. The resonance frequency varies with the excitation orientation for the split natural frequencies but it remains constant for the repeated natural frequencies. Potential applications of the spatial-dependent resonance mode and frequency are presented.

1. Introduction

Rotationally periodic structures (RPS) are widely used in engineering applications, for instance the ring structure in gyroscope, stators in ultrasonic and induction motors, and ring gear in planetary gear trains. The RPS consist of an axisymmetric ring (or plate) substrate and periodic features including external support, stator tooth and gear tooth, etc. The vibration in the presence of periodic features can deviate from those of axisymmetric structure. A bulk of publications have examined the free and traveling wave responses. This work focuses on the standing wave response in particular the difference between the two types of responses to the traveling wave excitation and its mathematically equivalent standing wave excitation.

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Previous studies have clearly revealed the rules governing the natural modes and natural frequencies of axisymmetric structure and RPS. For an axisymmetric structure, there exist two orthogonal degenerate nodal diameter modes with the same natural frequency. Such mode pairs have arbitrary spatial orientation [1], as shown in Fig. 1(a). However, the doubly-degenerate modes for the RPS have preferred orientations corresponding to the split natural frequencies, provided that two times of the wavenumber is integer multiple of the feature count [2–6], as shown in Fig. 1(b). The preferred orientations can be determined by Rayleigh's principle [2]. If the condition is not satisfied, the doubly-degenerate modes will associate with the repeated natural frequencies with arbitrary orientations. Note that Fig. 1(b) takes mass particle as example to illustrate the natural modes, though imperfections from manufacturing errors can also cause similar behavior [6–8]. The above results are obtained by using analytical [2–4], finite element [9–12], and experiment [9,13,14] methods based on classical free vibration theory.

In the case of forced vibration, the axisymmetric structure and RPS exhibit different behaviors. For the axisymmetric structure subjected to single standing wave excitation, the orientation of the nodal diameter mode solely depends on the external excitation, and the resonance frequency is spatial-invariant wherever the excitation orientation is [1]. For the RPS, by contrast, apart from the conventional mode orientations in Fig. 1(b), there can exist other orientations for the resonance mode, just like the axisymmetric structure, as shown in Fig. 1(c). If this is true, the contribution of the mass effect to the modes in Fig. 1(c) is different from those in Fig. 1(b) due to the distinct positions of the mass particles, and thus the resonance frequencies are changed. The "resonance" is used here since the modes in Fig. 1(c) are not the conventional natural modes any more. The existences of the resonance modes and associated resonance frequencies need to be verified in particular in an experimental fashion.

By contrast, for the RPS subjected to two standing wave excitations, the superposition of natural modes were usually employed to calculate response. Chang and Wickert [4] obtained the response by superposing the two standing waves corresponding to the natural modes. Due to the natural frequency splitting, the superposition results in a distorted traveling wave. They verified them through an experiment based on a rotating plate stimulated by airflow from nozzles [9]. The distorted wave can also be approached by using the perturbation and/or superposition [15,16]. In this respect, the perturbation method is preferred because it is convenient and accurate enough for RPS with small deviation from axisymmetry [17]. Exact method based on Floquet-bloch theorem [18–20] can also be applied to periodic structures with complex coupling between adjacent periodic elements.

As the most conventional treatment, the standing wave pair being in phase with the natural modes is generally used in the superposition method [4]. However, a traveling wave excitation can be equivalently decomposed into other standing wave pairs with different spatial phases. For this decomposition, the final results can also be different, but the study in this respect is scarce. This difference is roughly explained through the resonance modes in Fig. 1(c). That is, the response to the standing wave pairs can be obtained by superposing the resonance modes instead of the natural modes any more. Of course, further research based on the natural and resonance modes is very essential to verify the possible difference between the responses to the two types of excitations.

This work examines the spatial-dependent resonance mode and resonance frequency through analytical and experimental methods. As a representative case, a circular ring on an elastic foundation with equispaced small mass blocks is employed. An analytical model is developed in Section 2. The free and forced responses are formulated by using the perturbation-superposition method [3,24], and the relationships between the natural and resonance modes are detailed in Section 3. Comparisons between the two types of responses based on the natural mode and the proposed resonance mode are made in Section 4. In Section 5, a test rig is designed and fabricated to verify the existence of resonance mode and the variation in resonance frequency with the excitation orientation. In Section 6, the engineering applications of the spatial-dependent resonance mode and frequency are discussed for the traveling wave ultrasonic motor and other periodic structures.

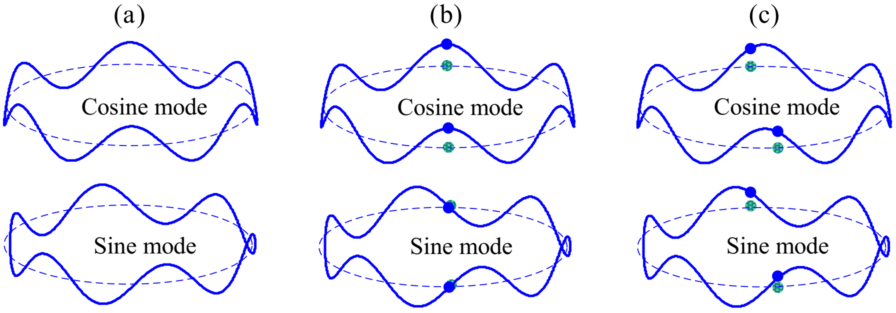

Fig. 1. Vibration modes of axisymmetric ring and RPS with two mass particles and six wavenumbers, where (a) natural modes of axisymmetric ring, (b) natural modes of RPS, and (c) resonance modes of RPS.