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Partial quadratic eigenvalue assignment in vibrating structures using receptances and system matrices☆

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In this paper, we consider the partial quadratic eigenvalue assignment problem (PQEAP) in vibration by active feedback control. Based on the receptance measurements and system matrices, we propose a constructive method for solving PQEAP, where we only need to solve a small linear system and only a few undesired open-loop eigenvalues with associated eigenvectors are needed. Our method is designed for both single-input and multiple-input vibration controls of vibrating structures. The real form of our method is also presented. Numerical tests show that our method is effective for constructing a solution to PQEAP with both single-input and multiple-input vibration controls.

1. Introduction

In structural dynamics and vibration, by using the finite element technique, the dynamics of vibrating structures (e.g., bridges, cars, buildings, and crafts, etc) are often modeled as a system of second-order differential equations of the form:

$$M\ddot{\mathbf{x}}(t) + C\dot{\mathbf{x}}(t) + K\mathbf{x}(t) = \mathbf{h}(t), \tag{1}$$

where $\mathbf{x}(t)$ means the displacement vector, $\mathbf{h}(t)$ denotes the external force vector, and the real $n \times n$ matrices M, C, K are mass, damping and stiffness matrices respectively. The natural frequencies and mode shapes of vibrating structures are determined by the eigenvalues and eigenvectors of the quadratic matrix pencil [4,5,19,20,36]:

$$P(\lambda) \equiv \lambda^2 M + \lambda C + K.$$

Vibrating structures often show undesired vibrations such as resonance especially when they encounter some dangerous external force $\mathbf{h}(t)$ (e.g., earthquake). A classical method is using passive damping. However, this method has severely practical limitations. A more modern method is using Active Vibration Control (AVC). To implement the AVC technique, we often need to get the feedback matrices such that a few undesired natural frequencies are replaced by the prescribed ones while the remaining large number of natural frequencies and mode shapes are kept unchanged.

Mathematically, the feedback vibration control can be described as follows. Suppose that the control external force has the form of

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$$\mathbf{h}(t) = B\mathbf{u}(t), \quad \mathbf{u}(t) = F^T\dot{\mathbf{x}}(t) + G^T\mathbf{x}(t), \tag{2}$$

where B is a given real n-by-m control matrix, $\mathbf{u}(t)$ is the associated control m-vector, and the real n-by-m matrices F and G are called feedback matrices. The feedback vibration control aims to find the feedback matrices $F, G \in \mathbb{R}^{n\times m}$ such that the closed-loop pencil

$$P_c(\lambda) \equiv \lambda^2 M + \lambda(C - BF^T) + (K - BG^T)$$

has the eigenvalues $\mu_1, \ldots, \mu_p, \lambda_{p+1}, \ldots, \lambda_{2n}$, where $\mu_1, \ldots, \mu_p$ ($p \ll 2n$) are the prescribed ploes, which are assigned to replace the unwanted eigenvalues $\lambda_1, \ldots, \lambda_p$. In addition, the eigenvectors $\mathbf{x}_{p+1}, \ldots, \mathbf{x}_{2n}$ corresponding to the no spill-over eigenvalues $\lambda_{p+1}, \ldots, \lambda_{2n}$ are required to remain unchanged, where $\{\lambda_k, \mathbf{x}_k\}_{k=1}^{2n}$ denote the $2n$ eigenvalues and eigenvectors of the open-loop pencil $P(\lambda)$. This is the so-called Partial Quadratic Eigenvalue Assignment Problem (PQEAP).

There exists a large literature on the direct solution to the PQEAP for single-input and multi-input second-order vibration control systems. See for instance [9,12–16,28]. To improve stability of the closed-loop vibration systems and reduce the energy consumption and noise influence, some optimization-based methods are developed for feedback minimization and robustness of the PQEAP. See for instance [3,6–8,10,11,21–23,27]). All these methods use only the knowledge of system matrices M, C, and K and a few unwanted natural frequencies and corresponding mode shapes.

Recently, there are some work on pole assignment problem by using the method of receptances. One may refer to [24–26,29,30,32–35] for the related developments on the method of receptances in AVC. The advantage of the method of receptances is that an output feedback is obtained by using the measured receptances without the knowledge of the system matrices M, C, K. On the other hand, there are some work on the PQEAP by using the measured receptances and the system matrices M, C, K. See for instance [1,2,31].

In this paper, we propose a constructive method for the PQEAP by using the measured receptances and the system matrices. This is motivated by Ram and Mottershead [30]. In [30], a linear system-based parametric solution to the PQEAP was provided by using the measured receptances, where the additional $2n - p$ eigenpairs $\{\lambda_k, \mathbf{x}_k\}_{k=p+1}^{2n}$ are needed. To keep the no spill-over property of the PQEAP and use measurability of receptances, we propose a constructive method for the single-input and multi-input PQEAP by using the measured receptances and the system matrices M, C, K, where we only need to solve a small linear system. We also present the real reformulation of our method so that all the computation are implemented in real operations.

The main contributions of this paper are as follows: A new constructive method is proposed for the single-input and multi-input PQEAP.

(i) It need only a few undesired frequencies and mode shapes, which can be measured at a vibration laboratory or computed using state-of-the-art computational techniques.
(ii) It uses both the measured receptances and the system matrices M, C, K, which are readily available by finite element techniques to the vibrating structures [19].
(iii) The explicit parametric solution is provided by solving a small linear system with a $p \times p$ coefficient matrix.

Illustrative numerical examples are presented to demonstrate the effectiveness of the proposed method.

2. The PQEAP by receptances and system matrices

In this section, we propose a constructive method for solving the PQEAP by receptances and system matrices, where only a few unwanted open-loop eigenvalues with associated eigenvectors are required. Our method is described for both single-input and multiple-input vibration control of vibration structures.

In what follows, we assume that M, C and K are real symmetric with M being positive definite. Denote by A^T, $\overline{A}$, and A^H the transpose, the conjugate, and the conjugate transpose of a matrix A accordingly. Let I_n be the identity matrix of order n. Let $\mathbf{y}_k$ be the eigenvector of the closed-loop pencil $P_c(\lambda)$ corresponding to the eigenvalue μ_k for $k = 1, \ldots, p$. Suppose that $\{\mu_1, \ldots, \mu_p\} \cap \{\lambda_1, \ldots, \lambda_{2n}\} = \varnothing$ and $\{\lambda_1, \ldots, \lambda_p\} \cap \{\lambda_{p+1}, \ldots, \lambda_{2n}\} = \varnothing$. The control matrix B has full column rank, and $(P(\lambda), B)$ is partially controllable with respect to the eigenvalues $\lambda_1, \ldots, \lambda_p$, i.e.,

$$\mathrm{rank}([P(\lambda_i) \ \ B]) = n, \quad i = 1, \ldots, p.$$

Let

$$\Lambda_1 = \mathrm{diag}(\lambda_1, \ldots, \lambda_p), \quad \Lambda_2 = \mathrm{diag}(\lambda_{p+1}, \ldots, \lambda_{2n}), \quad \Lambda_c = \mathrm{diag}(\mu_1, \ldots, \mu_p), X_1 = [\mathbf{x}_1 \ \mathbf{x}_2 \ \cdots \ \mathbf{x}_p], \quad X_2 = [\mathbf{x}_{p+1} \ \mathbf{x}_{p+2} \ \cdots \ \mathbf{x}_{2n}],$$
$$Y_1 = [\mathbf{y}_1 \ \mathbf{y}_2 \ \cdots \ \mathbf{y}_p].$$

Without loss of generality, we assume that the two sets $\{\lambda_1, \lambda_2, \ldots, \lambda_p\}$ and $\{\mu_1, \mu_2, \ldots, \mu_p\}$ are closed under complex conjugate with

$$\mu_{2k-1,2k} = \mu_{kR} \pm \mathrm{i}\mu_{kI} \quad k = 1, \ldots, l; \quad \mu_k \in \mathbb{R} \quad k = 2\,l + 1, \ldots, p \quad (0 \le 2\,l \le p)\lambda_{2k-1,2k} = \lambda_{kR} \pm \mathrm{i}\lambda_{kI}, \quad k = 1, \ldots, t; \quad \lambda_k \in \mathbb{R},$$
$$k = 2t + 1, \ldots, p \quad (0 \le 2t \le p), \mathbf{x}_{2k-1,2k} = \frac{1}{\sqrt{2}}(\mathbf{x}_{kR} \pm \mathrm{i}\mathbf{x}_{kI}), \quad k = 1, \ldots, t; \quad \mathbf{x}_k \in \mathbb{R}^n, \quad k = 2t + 1, \ldots, p,$$

where $\mu_{kR}, \lambda_{kR} \in \mathbb{R}$, $0 \ne \mu_{kI} \in \mathbb{R}$, $0 \ne \lambda_{kI} \in \mathbb{R}$, $\mathbf{x}_{kR} \in \mathbb{R}^n$, $\mathbf{0} \ne \mathbf{x}_{kI} \in \mathbb{R}^n$, and $\mathrm{i}:=\sqrt{-1}$ is the imaginary unit. Define