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Short communication

Compressive detection of unknown-parameters signals without signal reconstruction

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ABSTRACT

A novel algorithm for detecting compressed signals in low Signal-Noise-Ratio (SNR) without signal reconstruction is proposed in this letter. When signals are projected in a sparse domain, the sparse vector with fixed position of non-zero elements is obtained and the non-zero elements obey Rician distribution. However, additive white Gaussian noise (AWGN) is not sparse in this transformed domain, and the weight vector element amplitudes follow Rayleigh distribution while the AWGN is projected in the field. Thus, the distribution of sparse vector element amplitudes (SVEA) is considered to design a detector for an unknown-parameters signal. In addition, the accumulation of sparse vector in a sparse domain solves the problem of low-SNR signal detection. Later, the performance of the proposed detector is studied, and computer simulations show that it can detect the signals with a probability of 95% under the conditions that $SNR = -28$ dB and compressive ratio $M/N = 0.15$. Furthermore, the receiver operating characteristic (ROC) theoretical and simulation curves are drawn.

1. Introduction

Different from classical Nyquist sampling theorem, Compressive Sensing (CS) provides a new promising method for signal processing, which transforms the original signal into a new sparse domain signal with less sampling measurements. Then, signals can be reconstructed via the compressive measurements with high probability [1,2]. CS is now reaching to maturity stage and lots of techniques especially in areas of image compression have been developed [3–5]. Based on CS theory, when signal is sparse in a transformed domain, a large amount of data is redundant. Hence, fewer measurements are required in the sparse domain. In the past several years, CS has been used in radar signal processing, wideband signal processing, *etc.* [6–8]. Originally, some researchers in the field of CS focus on reconstructing the compressed signals, detecting signals or estimating signal parameters [1,9].

Recently, compressive detection without signal reconstruction [7,10,11] has attracted more attentions. Instead of reconstructing signal completely, the authors partly reconstruct compressed signals and then detect the signal when SNRs are high [9]. Some researchers propose the compressive detection algorithms based on traditional matched filter methods [7,12]. It is inefficient for an unknown-parameter signal detection that all exact information of the signal must be provided before detection. In [10,11], a signal detection algorithm based on Bayesian method is proposed, which needs the prior probability information of the sparse signal. In addition, unknown parameters of sparse signal detection are proposed in [13,14]. Random subspace signal and agnostic signal following Gaussian distribution are studied. However, it is necessary to find the subspace basis matrix firstly. Next, the variance of the signal should be prior acknowledged. In [13], the authors mainly research the random subspace signal with unknown variance. But the determined subspace basis matrix for sparse signal limits the application of signal detection.

The main purpose of this letter is to detect compressed signal without signal reconstruction. For a given sparse signal, the position of the maximum non-zero elements is fixed in the sparse vector, which is a wonderful feature for signal accumulating in the sparse domain. Furthermore, the maximum non-zero element amplitude follows Rician distribution. When additive white Gaussian noise (AWGN) is projected in the transformed domain, the projection vector(PV) whose position of maximum non-zero element presents a uniform distribution can be determined and every element of PV obeys Rayleigh distribution. Based on characteristics of the sparse representation, the method of accumulating signals in a sparse domain and comparing the distribution of the maximum non-zero element in PV are considered for designing the detector. Moreover, the performance of the detector is formulated and verified by Monte Carlo computer simulations. The main advantage of

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this proposed detector is that it only needs no-parameter information to detect low-SNR signal without reconstructing.

The remaining of this letter is as follows: firstly, in Section 2, we build the signal model. Secondly, we introduce the proposed detector and analyse the performance of the proposed detector in Section 3. In Section 4, some simulations are provided to demonstrate the performance of the proposed detector. Finally, in Section 5, we make a conclusion.

2. Signal model

A sparse signal can be represented as follows

$$\mathbf{s} = \mathbf{\Psi x} \tag{1}$$

where $\mathbf{s}$ is the signal to be detected. $\mathbf{\Psi}$ represents a unit $N \times N$ basis matrix and $\mathbf{x} \in \mathbf{R}^{N \times 1}$ is the sparse vector with $K(K << N)$ non-zero elements. When $\mathbf{s}$ is projected in $\mathbf{\Psi}$, the positions of non-zero elements are fixed and the amplitudes of non-zero elements represent main 'energy' of the signal. Thus, the method of accumulation of the sparse vector element amplitudes (SVEA) is considered in the sparse field.

The problem of traditional signal hypothesis detection model [13] is obtained

$$\begin{cases} H_0 : \mathbf{z}_n = \mathbf{w}_n, & n = 1, \dots, J \\ H_1 : \mathbf{z}_n = \mathbf{s}_n + \mathbf{w}_n, & n = 1, \dots, J \end{cases} \tag{2}$$

where $\mathbf{z}_n$ is the received signal polluted by AWGN, $\mathbf{w}_n \sim N(0_{N,1}, \sigma_n^2 I_N)$ represents the received noise with a series of observations $n = 1, \dots, J$. A great amount of observation data is processed for uncompressed signal, which is difficult to be realized by hardware and waste of resources. Based on CS theory, when signal is sparse in a transformed domain, fewer measurements are required in the sparse domain. Therefore, the compressive sensing hypothesis-testing problem is described from Eqs. (1) and (2)

$$\begin{cases} H_0 : \mathbf{y}_n = \mathbf{\Phi w}_n, & n = 1, \dots, J \\ H_1 : \mathbf{y}_n = \mathbf{\Phi}(\mathbf{\Psi x}_n + \mathbf{w}_n), & n = 1, \dots, J \end{cases} \tag{3}$$

where $\mathbf{\Phi} \in \mathbf{R}^{M \times N}$ is a real random Gaussian measurement matrix with i.i.d elements having zero mean and unit variance. We can acquire a sequence of $\mathbf{y}_n$ and define PV as follows

$$\mathbf{T}_n = \mathbf{\Psi \Phi}^H \mathbf{y}_n, \quad n = 1, \dots, J \tag{4}$$

Then, we obtain a matrix $\mathbf{A} \in \mathbf{R}^{N \times J}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_2 & \cdots & \mathbf{T}_J \end{bmatrix} \tag{5}$$

where $\mathbf{A}$ represents a set of PV, and $\mathbf{T}_j = \begin{bmatrix} \alpha_j(1) & \alpha_j(2) & \cdots & \alpha_j(N) \end{bmatrix}^T$.

3. Compressive detection

3.1. Proposed method

The summation of non-zero elements in the sparse vector represents main 'energy' of signal. AWGN is not sparse and projected in $\mathbf{\Psi}$ still following Gaussian distribution [13]. If there is AWGN corrupted the signal, the real and imaginary parts of $N - K$ zero elements of $\mathbf{x}_n$ obey the following distribution

$$\mathrm{Re}\left(\alpha_j(n)\right), Im\left(\alpha_j(n)\right) \sim N\left(0, \frac{\sigma_n^2 \mathbf{\Phi}^H \mathbf{\Phi \Psi}^H \mathbf{\Psi \Phi}^H \mathbf{\Phi}}{2}\right) \tag{6}$$

where $j = 1, \dots, N - K$ and σ_n^2 means the variance of every element in PV. Next, we define $\sigma_\Delta^2 = \frac{\sigma_n^2 \mathbf{\Phi}^H \mathbf{\Phi \Psi}^H \mathbf{\Psi \Phi}^H \mathbf{\Phi}}{2}$ and the maximum value of sparse vector $\mathbf{x}_n$ is represented as α_i. The distribution of α_i is described as

$$\begin{cases} \mathrm{Re}\left(\alpha_i(n)\right) \sim N\left(\sqrt{\sigma_{sr}^2}\mathbf{\Phi^H \Phi}, \sigma_\Delta^2\right) \\ \mathrm{Im}(\alpha_i(n)) \sim N\left(\sqrt{\sigma_{si}^2}\mathbf{\Phi^H \Phi}, \sigma_\Delta^2\right) \end{cases} \tag{7}$$

where $\sqrt{\sigma_{sr}^2}$ and $\sqrt{\sigma_{si}^2}$ stand for the amplitude of $\mathrm{Re}(\alpha_i(n))$ and $\mathrm{Im}(\alpha_i(n))$. $\mathbf{\Phi^H \Phi} \sim \chi_M^2$ follows chi-squared distribution with M degrees of freedom [13], and $\mathbf{\Psi}^H \mathbf{\Psi} = \mathbf{I}_N$. Thus, the mean value of $\alpha_i(n)$ is

$$\begin{cases} \mu_{sr} = \sqrt{\sigma_{sr}^2}\mathbf{\Phi^H \Phi} = M\sqrt{\sigma_{sr}^2} \\ \mu_{si} = \sqrt{\sigma_{si}^2}\mathbf{\Phi^H \Phi} = M\sqrt{\sigma_{si}^2} \end{cases} \tag{8}$$

The signal energy $\sigma_s^2(\sigma_s^2 \approx \varepsilon * (\sigma_{sr}^2 + \sigma_{si}^2))$, where ε is the maximum weight coefficient when signal 'energy' distributes in a sparse domain. For convenience, we define the variance of $\alpha_j(n)$ and $\alpha_i(n)$ as

$$\sigma^2 = \sigma_n^2 * diag(\mathbf{\Phi^H \Phi \Psi^H \Psi \Phi^H \Phi}) = \sigma_n^2 * M(M + N) \tag{9}$$

where $diag(\mathbf{w})$ represents diagonal matrix. We can combine Eqs. (6) and (7) as below

$$\begin{cases} Re(\alpha_j(n)), Im(\alpha_j(n)) \sim N\left(0, \sigma^2\right) \\ Re(\alpha_i(n)) \sim N\left(\mu_{sr}, \sigma^2\right) \\ Im(\alpha_i(n)) \sim N\left(\mu_{si}, \sigma^2\right) \end{cases} \tag{10}$$

Then, denote a new vector $\mathbf{B} \in \mathbf{R}^{1 \times N}$ for convenience, which represents the accumulation of PV in the sparse domain.

$$\mathbf{B} = \left|mean\left(\mathbf{A}^T\right)\right| = \begin{bmatrix} \widehat{\alpha}_1 & \dots & \widehat{\alpha}_N \end{bmatrix}$$
$$= \begin{bmatrix} \left|\frac{1}{J}\sum_{k=1}^{J}\alpha_k(1)\right| & \dots & \left|\frac{1}{J}\sum_{k=1}^{J}\alpha_k(N)\right| \end{bmatrix} \tag{11}$$

where $\widehat{\alpha}_1, \dots, \widehat{\alpha}_N$ are i.i.d with J times observations as well.

Next, we assume a new vector $\mathbf{D} \in \mathbf{R}^{(N-K) \times 1}$ to describe the $N - K$ zero elements of $\mathbf{x}_n$ mentioned above.

$$\mathbf{D} = \begin{bmatrix} \widehat{\alpha}_1 & \dots & \widehat{\alpha}_j & \dots & \widehat{\alpha}_{N-K} \end{bmatrix} \tag{12}$$

The variance of $\widehat{\alpha}_j$ is

$$\widehat{\sigma}_n^2 = \frac{\sigma_n^2 * diag(\mathbf{\Phi^H \Phi \Psi^H \Psi \Phi^H \Phi})}{2J} = \frac{\sigma^2}{2J} \tag{13}$$

The amplitude of $\widehat{\alpha}_j$ obeys the Rayleigh distribution and the maximum element in sparse vector $\widehat{\alpha}_i$ is follows the Rician distribution [15], and the probability density function (PDF) can be described as follows respectively

$$\begin{cases} f_{\widehat{\alpha}_j}(\alpha) = \frac{\alpha}{\widehat{\sigma}_n^2}\exp\left(-\frac{\alpha^2}{2\widehat{\sigma}_n^2}\right) \\ f_{\widehat{\alpha}_i}(\alpha) = \frac{\alpha}{\widehat{\sigma}_n^2}\exp\left(-\frac{1}{2\widehat{\sigma}_n^2}(\alpha^2 + \frac{M\sigma_s}{\varepsilon})\right) I_0\left(\frac{M\sigma_s}{\varepsilon\widehat{\sigma}_n^2}\right) \end{cases} \tag{14}$$

where $I_n(\bullet)$ denotes the modified Bessel function of nth [15]. From Eqs. (10) and (14), the mean value of $\widehat{\alpha}_i$ should be the largest than that of every element of $\mathbf{D}$. So that the signal is considered to be represented if $\bigcap_{j=1}^{N-K}\left(\widehat{\alpha}_i > \widehat{\alpha}_j\right)$. The probability of $P_{\widehat{\alpha},D}$ is expressed as

$$P_{\widehat{\alpha},D} = P\left(\bigcap_{j=1}^{N-K}\left(\widehat{\alpha}_i > \widehat{\alpha}_j\right)\right) \tag{15}$$