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A new method for parameter estimation of high-order polynomial-phase signals

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ABSTRACT

Parameter estimation of a high-order polynomial phase signal (PPS) is considered in this paper. We propose a method to estimate phase parameters more efficiently and accurately. In the proposed method, we define an operator referred to as non-uniform sampled reducing-order operator (NURO) to reduce the order of a polynomial phase by half, when the order of PPS is even. By combined using NURO and phase differentiation (PD) operators, the PPS order is reduced to one, i.e., the PPS degenerates into a complex sinusoid. Then, the parameter estimation can be done by jointly using fast Fourier transform (FFT) and one-dimensional search. Compared with the traditional methods, the reducing-order procedure in the proposed method has lower-order nonlinearities. Simulation results show that the proposed method outperforms the hybrid CPF-HAF and HAF in both mean square error (MSE) and the threshold when the PPS order is higher than 5.

1. Introduction

Polynomial phase signal (PPS) has widely application in the parameter estimation and target detection in radar, sonar, communication, and various nature signal analysis [1–7]. To estimate the parameters of PPS, the maximum likelihood (ML) estimation was proposed in 1960s. Although it performs well in the sense of mean square error (MSE), it suffers a heavy computational burden due to the multidimensional search. Therefore, methods with low computational burden attract more and more attention. The high-order ambiguity function (HAF) [2], which is realized by a one-dimensional search, develops a phase differentiation (PD) technique to reduce the PPS order. By applying PD once, PPS order can be decreased by one. However, as to high-order PPS, many times of using PD leads to high-order nonlinearities, which increases the number of noise influenced terms. Therefore, the HAF algorithm has a poor performance at low signal-to-noise ratio (SNR).

To estimate the third order phase parameter, some time-frequency rate representations are proposed, for example, the cubic phase function (CPF) [3], the high resolution time-frequency rate representation (HR-TFRR) and their extensions [4,5]. By combining the CPF and HAF, the hybrid CPF-HAF is proposed in [6] to estimate parameters of high-order PPSs. Compared with the HAF, the hybrid CPF-HAF algorithm requires two PDs less which leads to relatively lower MSE and lower SNR threshold. Although the hybrid CPF-HAF improves the estimate performance, the order of nonlinearities grows exponentially with the increase of the PPS order.

Expect the hybrid CPF-HAF algorithm, the quasi-maximum-likelihood (QML) estimator and its extensions [8–12] are proposed to improve the performance at low SNR. By utilizing the robustness of the short-time Fourier transform (STFT) to noise influence, the QML based algorithm achieves a lower SNR threshold and a lower MSE than the HAF. However, the improvement is achieved at the cost of greatly increased computational complexity.

In this paper, we propose a new method to estimate the phase parameters of a high-order PPS more efficiently and accurately. In the proposed method, we define the kernel in non-uniform sampled cubic phase function (NU-CPF) [13,14] as a reducing-order operator referred to as non-uniform sampled reducing-order operator (NURO). When the PPS order is even, NURO can decrease the PPS order by half. Utilizing this property, we perform a multistep procedure combining the PD and NURO operator to reduce the order of the PPS. Instead of using PD repeatedly in the HAF based approach, this procedure selects and uses either PD or NURO operator in each stage. When the PPS order is odd, we perform PD and reduce the PPS order by one. On the other hand, if the PPS order is even, we perform NURO and reduce the PPS order by half. In this manner, by using the PD or NURO repeatedly, we can transform a high-order PPS into a complex sinusoid with fewer bilinear operations i.e., involving lower-order kernel nonlinearities. Since the frequency of the sinusoid is proportional to the highest-order phase

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parameter, the estimate of highest-order phase parameter can be done efficiently by jointly using fast Fourier transform (FFT) and one dimensional search. The proposed algorithm outperforms traditional estimate algorithms for high-order PPSs in MSE and signal-to-noise (SNR) threshold.

2. PD and NURO operator

2.1. PD operator

Consider a discrete PPS defined as

$$x(n) = s(n) + w(n), -N/2 \le n \le N/2$$
$$s(n) = b_0 \exp\left(j\phi(n)\right) = b_0 \exp\left(j\sum_{i=0}^{P} a_i (n\Delta)^i\right), \tag{1}$$

where $\phi(n)$ is the signal phase, P is the order of the polynomial in the signal phase, Δ is the sampling interval, w is complex zero-mean white Gaussian noise with variance σ^2, and $\{b_0, a_0, a_1, a_2, ..., a_P\}$ are real-valued parameters of the PPS. The length of the sequence is $N+1$, where N is even. The SNR is defined as $\text{SNR} = {b_0}^2/\sigma^2$. In this paper, we assume b_0 and a_0 are known constant. The problem is to estimate parameters $\{a_1, a_2, ..., a_P\}$ from $x(n)$.

The PD operator is used in the HAF to decrease the order of the polynomial in signal phase. It is defined in [2] as

$$\begin{aligned} PD_\tau[s(n)] &= s^*(n-\tau)s(n+\tau) \\ &= b_0^2 \exp\left(j\left(2\sum_{p=0}^{P-1} n^p \sum_{l=1}^{\lceil (P-p+1)/2\rceil} \frac{(p+2l-1)!}{p!(2l-1)!}\Delta^{p+2l-1}\tau^{2l-1} a_{p+2l-1}\right)\right) \\ &= b_0^2 \exp\left(j\left(\varphi_0(n) + \theta_1 n^{P-1}\right)\right), \end{aligned} \tag{2}$$

where $\theta_1 = 2P\Delta^P\tau a_P$, $\varphi_0(n)$ is a polynomial including lower-power terms of n, and $\lceil \cdot \rceil$ denotes the rounding up operation. From (2) we can see that the resulting signal phase is a polynomial with respect to n, and the order of the PPS is decreased by one. Knowing that the original coefficient of the highest-power term n^P in the phase of $s(n)$ is $\theta_0 = \Delta^P a_P$, the coefficient is changed by PD operator according to

$$\theta_1 = 2P\tau\theta_0. \tag{3}$$

In the HAF algorithm, the property of PD is utilized to reduce the order of the polynomial in the signal phase. The HAF algorithm can be described as follows. First, perform PD repeatedly until the PPS is transformed into a complex sinusoid:

$$\begin{aligned} PD^1_{\tau_1}[x(n)] &= x^*(n-\tau_1)x(n+\tau_1) \\ PD^2_{\tau_1\tau_2}[x(n)] &= \left\{PD^1_{\tau_1}[x(n-\tau_2)]\right\}^* PD^1_{\tau_1}[x(n+\tau_2)] \\ &\vdots \\ PD^{P-1}_{\tau_1\tau_2\ldots\tau_{P-1}}[x(n)] &= \left\{PD^{P-2}_{\tau_1\tau_2\ldots\tau_{P-2}}[x(n-\tau_{P-1})]\right\}^* PD^{P-2}_{\tau_1\tau_2\ldots\tau_{P-2}} \\ &\quad \times [x(n+\tau_{P-1})]. \end{aligned} \tag{4}$$

Then the highest-order parameter can be estimated by

$$\hat{a}_P = \frac{\arg\max_{\Omega} \left|\sum_n PD^{P-1}_{\tau_1\tau_2\ldots\tau_{P-1}}[x(n)]\exp(-j\Omega n)\right|}{2^{P-1}\Delta^P P! \prod_{i=1}^{P-1}\tau_i}. \tag{5}$$

The next lower-order parameter a_{P-1} can be estimated by performing the same procedure on the demodulated signal $x(n)\exp(-ja_P\Delta^P n^P)$. From (4) it can be seen that for a P th-order PPS, the HAF algorithm requires $P-1$ times of PD, i.e., the HAF has a kernel of 2^{P-1} th-order nonlinearities, where the order of nonlinearities equals to the number of signal factors in the product kernel [14]. When estimating the parameters of high-order PPSs, the order of kernel nonlinearities is very high for HAF algorithm, which leads to a high SNR threshold [12,15].

2.2. NURO operator

For third order PPS, a technique called the CPF is proposed in [3]. It is defined by

$$CPF(n,\Omega) = \sum_k x(n+k)x(n-k)\exp(-j\Omega k^2). \tag{6}$$

The CPF has an interpretation as a time-frequency rate representation. Thus, the third and second-order parameters of the PPS can be estimated by evaluating the function at two different instants:

$$\hat{a}_3 = \frac{\arg\max_\Omega |CPF(n_1,\Omega)| - \arg\max_\Omega |CPF(0,\Omega)|}{2n_1\Delta}, \tag{7}$$

$$\hat{a}_2 = \arg\max_\Omega |CPF(0,\Omega)|. \tag{8}$$

To improve the performance of the non-linear technique based algorithm at low SNR, the idea of reducing the order of nonlinearities is brought up in [18] and [19]. Basing on CPF and PD operator, the hybrid CPF-HAF is also proposed in [6]. This algorithm applies $P-3$ times of PD to reduce the order of polynomial in phase to 3, and then CPF is used to estimate the parameters. The resulting order of kernel nonlinearities is 2^{P-2}. Compared with HAF, it uses two PDs less, therefore, the lower-order kernel nonlinearities leads to a better performance in the MSE and SNR threshold [6]. However, the order of kernel nonlinearities still grows exponentially with the increase of the PPS order. Thus, the SNR threshold is still very high when estimating parameters of high-order PPS.

It can be seen from (6) that, in the CPF, a bilinear operator is applied to the PPS, which can be written as

$$CP[s(n)] = s(n-m)s(n+m). \tag{9}$$

Substituting a Pth-order PPS described in (1) into (9) yields

$$\begin{aligned} CP[s(n)] &= b_0 \exp\left(j\sum_{i_1=0}^{P} a_{i_1}\Delta^{i_1}(n+m)^{i_1}\right) \\ &\quad \times b_0 \exp\left(j\sum_{i_2=0}^{P} a_{i_2}\Delta^{i_2}(n-m)^{i_2}\right) \\ &= b_0^2 \exp\left(j\sum_{i_1=0}^{P} 2a_{i_1}\Delta^{i_1}\sum_{l=0}^{\lfloor i_1/2\rfloor}\binom{i}{2l} n^{i_1-2l}m^{2l}\right) \\ &= b_0^2 \exp\left(2j\sum_{l=0}^{\lfloor P/2\rfloor}\sum_{i_1=2l}^{P}\frac{i_1!}{(i_1-2l)!(2l)!}a_{i_1}\Delta^{i_1}n^{i_1-2l}m^{2l}\right) \\ &= b_0^2 \exp\left(j\sum_{l=0}^{\lfloor P/2\rfloor}\frac{2m^{2l}}{(2l)!}\sum_{i_1=2l}^{P}\frac{i_1!}{(i_1-2l)!}a_{i_1}\Delta^{i_1}n^{i_1-2l}\right) \\ &= b_0^2 \exp\left(j\sum_{l=0}^{\lfloor P/2\rfloor}\frac{2\phi^{2l}(n)}{(2l)!}m^{2l}\right), \end{aligned} \tag{10}$$

To reduce the computational complexity of CPF, the NU-CPF is proposed in [13], where the bilinear operator is modified so that the CPF can be performed by using FFT. We name the modified kernel in the NU-CPF as the NURO operator, which is given by

$$NUCP[s(n)] = s(n-\sqrt{cm})s(n+\sqrt{cm}). \tag{11}$$

Here, we exploit its usage of reducing PSS order. Similar to the derivation in (10), applying the NURO operator to a P th-order PPS