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Extreme-point weighted mode decomposition

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ABSTRACT

empirical mode decomposition (EMD) is an effective method for nonlinear and nonstationary signal analysis. In this paper a new signal decomposition method termed extreme-point weighted mode decomposition (EWMD) is proposed for improving the accuracy of EMD. In EWMD method, an newly intrinsic mode function (IMF) with physically meaning is defined to overcome the drawback of EMD that constructs mean curve by interpolating local extreme-points. Based on that, a new mean curve is constructed by using the weighting values of adjacent extreme-points for sifting process. Also a new IMF criterion closely related to its definition is established. We have deeply studied and compared the proposed method with EMD method by analyzing synthetic and mechanical vibration signals and the results show the superiority of proposed method in IMF accuracy, decomposition capability and orthogonality.

1. Introduction

Hilbert-Huang Transform (HHT) introduced by Huang et al. has been proved an effective and powerful time-frequency analysis method [1–4]. HHT contains two parts, i.e. empirical mode decomposition (EMD) and Hilbert transform (HT). EMD adaptively decomposes a complicated signal into numbers of intrinsic mode functions (IMFs) without priori determined basis functions that exists in the traditional Fourier transform or wavelet analysis methods. In original EMD [1], the mean curve is defined as average of upper and lower envelopes that respectively are defined by using cubic splines to interpolate all local maxima and minima of time series. Undoubtedly, the mean curve construction in EMD is the most crucial link and many improvements in mean curve construction have been made in recent years and some new adaptive signal decomposition methods were proposed successively. Typically, Chen et al. [5] took the moving average of a linear combination of cubic B-splines through the local extrema to construct mean curve. Pegram et al. [6] used the rational splines replaced cubic splines in the envelope-fitting procedure. The local mean decomposition method proposed by Smith [7] in nature also is a variant of EMD in mean curve construction. Intrinsic time-scale decomposition (ITD) proposed by Frei and Osorio [8] constructs baseline by using linear transform of original data directly and the remaining process is very similar to EMD method. Local characteristic-scale decomposition (LCD) proposed by Zheng, etal [9] constructS the mean curve by using cubic spline to replace the linear transform of ITD and get a better decomposition result than ITD. Daubechies et al [10] proposed an EMD like tool called synchrosqueezed wavelet transform (SWT) by combining the classical WT analysis and time-frequency rearrangement technique for non-stationary data analysis [11,12]. However, SWT method deeply depends on the wavelet transform and in nature is not a fully adaptive wavelet transform. Another variational method for mode decomposition is the work of Hou etal. [13,14], where the authors propose to minimize a function which is build on some regularity assumptions about the different components and uses three order total variation priors. However, the proposed algorithm is somewhat sensitive to noise and the decomposing process is time-consuming. Based on wavelet transform, an empirical wavelet transform (EWT) was recently proposed by Gilles [15,16]. However, EWT is very sensitive to the partition of Fourier spectrum. Variational mode decomposition proposed by Dragomiretskiy and Zosso also depends on the default setting of mode number and other parameters [17].

In this paper, we make our effort on modification of EMD method. In EMD method, the cubic spline functions are respectively used to interpolate all local maximal and minimal values, which will cause envelope overshooting or undershooting when the adjacent maximal values (or minimal values) have large amplitude differences. Therefore, it is necessary to construct the mean curve in a new way. In this paper a new signal decomposition method termed extreme-point weighted mode decomposition (EWMD) is proposed and studied. In EWMD method, the mean points locating at new mean curve are constructed by a weighting method of three adjacent extreme points. Then a cubic spline function is employed to interpolate these mean points that generally have small and gradual amplitude differences, which can re-

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strain the overshooting or undershooting of interpolation to some extent. Also in EWMD we introduce the weight factor to adjust the mode decomposition of signals with time varying amplitudes.

The rest of this paper is organized as follows. In Section 2 we propose a new signal decomposition method termed extreme-point weighted mode decomposition (EWMD). The comparisons of EWMD with EMD and their decomposition capacity are given in Section 3. The influence of weight factor on EWMD is studied in Section 4. Application analysis of the EWMD method is given in Section 5 and conclusions are given in the final section.

2. Extreme-point weighted mode decomposition

In EMD method, if a function that satisfies the following two conditions will be taken as an intrinsic mode function (IMF): (1) in the whole data set, the number of extrema and the number of zero-crossing must either equal or differ at most by one; and (2) at any point, the mean value of upper and lower envelopes defined by using cubic spline interpolating all local maxima and local minima is zero.

The first condition is similar to the traditional narrow band requirements for a stationary Gaussian process. The second condition modifies the classical global requirement to a local one. It is necessary so that the instantaneous frequency will not have the unwanted fluctuations induced by asymmetric wave forms. Ideally, the requirement should be that the local mean of the data equals zero. However, the local mean involves a “local time scale” to compute the mean for non-stationary data, which is impossible to define [1]. In EMD method, the second condition in fact also has another meaning, i.e. it in fact is the final target and a reflection of sifting process of EMD. The target of sifting is separating the mean curve from original data until the residue signal with zero mean curve. Therefore, the definition of second condition is very important. However, the used cubic spline function in EMD will cause envelope overshooting or undershooting when adjacent maximal (or minimal) values have large amplitude differences. By introducing the weight factor and using a weighting of three adjacent extreme points, the EWMD method is proposed to enhance the performance of EMD. Here the second condition is replaced by that the weighted averages of any adjacent three extreme points are zero. The new condition is a local definition to limit the global characteristic.

For an arbitrary complex signal $X(t)$, the proposed EWMD method is described as follows.

(1) Let $r_0(t) = X(t)$, $h_l(t) = r_i(t)$, $i = 1$, $l = 1$;

(2) Identify all local extreme (maxima and minima) points of $h_{l-1}(t)$ and note them as (τ_k^l, X_k^l), $k = 1, 2, \ldots, m$. Define m_k^l and A_k^l as

$$m_k^l = \frac{X_{k-1}^l + \alpha X_k^l + X_{k+1}^l}{4} \tag{1}$$

and

$$A_k^l = \frac{\left|X_{k-1}^l\right| + \alpha\left|X_k^l\right| + \left|X_{k+1}^l\right|}{4} \tag{2}$$

where $k = 2, 3, \ldots, m$-1. By using boundary extending methods, such as mirror extension or mirror-symmetric extension [18,19], we can obtain m_1^l, m_m^l and A_1^l, A_m^l.

(3) Use cubic spline function to interpolate all (τ_k^l, m_k^l) and (τ_k^l, A_k^l) points ($k = 1, 2, \ldots, m$), respectively and correspondingly we get the mean curve $m_{l-1}(t)$ and envelop curve $A_{l-1}(t)$of $h_{l-1}(t)$.

(4) Separate mean curve $m_{l-1}(t)$ from $h_{l-1}(t)$ with residue $h_l(t)$ obtained, i.e.

$$h_l(t) = h_{l-1}(t) - m_{l-1}(t) \tag{3}$$

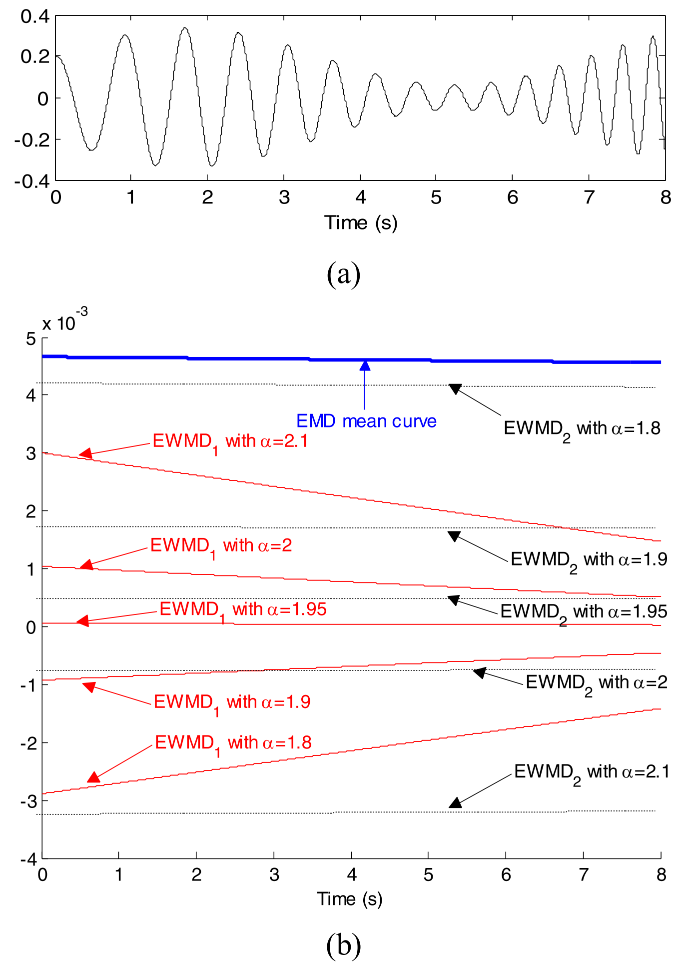

Fig. 1. A typical IMF signal and its mean curves estimated by different methods (a) A typical IMF signal and (b) its mean curves estimated by different methods.

(5) Let $l = l + 1$ and repeat steps (2) to (4) until $h_{l-1}(t)$ is an IMF and we deem $h_{l-1}(t)$ as an IMF and note $IMF_i = h_{l-1}(t)$.
Here, we define $\theta_{l-1}(t) = |m_{l-1}(t)/A_{l-1}(t)|$, if $\theta_{l-1}(t) < \theta_1$ for some prescribed fraction $(1 - \alpha)$ of the total duration, while $\theta_1 \le \theta_{l-1}(t) < \theta_2$ for the remaining fraction, i.e. $m_{l-1}(t)$ tends to zero.

(6) Let $r_i(t) = r_{i-1}(t) - IMF_i$ and $i = i + 1$. Implement steps (2) to (6) until that $r_i(t) = r_0(t) - IMF_1 - IMF_2 - \cdots - IMF_i$ is a monotonic function or a function with no more than three extreme points. Then the original data $X(t)$ is decomposed as $X(t) = \sum_{i=1}^{n} IMF_i + r_n(t)$.

There are several notes about the proposed method. Firstly, in step (2), we introduce the weight factor α. Generally, we set $\alpha = 2$. In order to illustrate the influence of α on construction of mean curve, without loss of generality, we consider a typical AM-FM signal shown in Fig. 1(a), which meets the definition of IMF and is an IMF. So its mean curve should be zero. For comparison purpose, we compute the mean curves using EMD, $EWMD_1$ and $EWMD_2$ ways with different weight factor α and the results are given in Fig. 1(b). From Fig. 1(b) it can found that the amplitude of mean curve obtained by EMD is about 0.0045, which is larger than that of all mean curves obtained by $EWMD_1$ and $EWMD_2$ for different weight factor. Especially, when the weight factor $\alpha = 1.95$, the amplitude of mean curve obtained by $EWMD_1$ is about 4×10^{-5} and more close to zero than that of EMD method. This example indicates the influence of weight factor on mean curve construction and superiority of EWMD method to EMD in mean curve construction. By exten-