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Short communication

# Pareto optimal layout of multistatic radar

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ABSTRACT

Layouts influence the performance of location accuracy greatly in multistatic radar. The layout in classical regular shape may not be an excellent choice. In this paper, Pareto optimal layout of multistatic radar is proposed based on dynamic game theory. The available volume associated with the Cramer-Rao lower bound (CRLB) of location accuracy is proposed as the utility metric to evaluate the performance of the layout. Simulation results show that different initial layouts may lead to different final layouts. Although these final layouts are different they are all Pareto optimal and similar in performance.



## 1. Introduction

The layout of multistatic radar has a significant influence on the performance of location accuracy. The optimization problem on the layout of multistatic radar consisting of $M$ transmitters and $N$ receivers is studied in this paper. The positions in the deployable area are possible positions for multistatic radar. A kind of layout means a permutation of $M+N$ possible positions corresponding to the $M$ transmitters and $N$ receivers. The aim is to choose the layout with an optimal performance of location accuracy. A direct idea is to enumerate all the choices in the problem and then choose the best, but this is not practical unless the quantities of sites and possible positions are very small. For example, with 3 transmitters, 3 receivers and 1000 possible positions, there are $10^{18}$ possible choices. So the direct enumeration is obviously impossible. The Cramer-Rao lower bound (CRLB) is often used as a performance metric for multistatic radar system. The CRLB is calculated in random sensor placements [1]. The choice of the layout affects the CRLB of location accuracy in a surveillance area.

Optimization of the layout has been studied in some classical regular shapes such as dual triangle [2] and circle [3] to avoid the scale expansion of possible positions. The possible positions are limited in these classical regular shapes. As a result, the improvement of accuracy performance is limited. In addition, the CRLBs of the specific target location or trajectory are selected as the performance metric of optimization in these pre-mentioned literatures, which means the optimization results can only adapt to these limited situations. Genetic algorithm [4–6] is studied to optimize the geometry or layout of the multistatic radar. But along with the extending of the problem, the genetic algorithm tends to get local extremum and unwanted early convergence. In some ways the optimization problem on layout is similar to the problem on sensor selection [7–10]. The problem on sensor selection is about how to achieve optimal selection in massive possibilities of sensor combinations [10]. The problem on sensor selection has been discussed as a knapsack problem in [9]. A heuristic arithmetic based on convex optimization has been proposed for resolving the problem in [10], and numerical experiments suggest that the performance is good in many cases, but there is no guarantee that the performance is always good in the heuristic arithmetic.

The layout optimization of multistatic radar [11,12] is a problem on resource allocation. The game theory [13] is often used to solve the problem on resource allocation. Pareto optimality [14–18] denotes the state that no player can promote the utility without damaging others' utility. Based on dynamic game theory, a method is proposed for optimizing the layout of multistatic radar in this paper. Receivers and transmitters are considered as the players. The utility achieved by each player is associated with the distribution of the CRLB in a surveillance area. The possible positions are the strategy space of the players. After the game running in several stages, we can get the Pareto optimal layout of multistatic radar.

In this paper, a novel method named Pareto optimal layout of multistatic radar is proposed. The available volume associated with the layout where the CRLB is smaller than an acceptable criterion is proposed as the performance metric of optimization in the proposed method. Therefore, the optimization results can adapt to a variety of situations. Simulation results show the effectiveness of the proposed method and also suggest that the proposed Pareto optimal layout of multistatic radar can be achieved in small computational complexity.

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The paper is organized as follows: Game theory model is presented in Section 2, CRLB and available volume is presented in Section 3, Pareto optimal layout is presented in Section 4. Simulation results and conclusions are presented in Section 5 and Section 6, respectively.

## 2. Game theory model

The multistatic radar consists of $M$ transmitters and $N$ receivers. Each receiver can receive the signals transmitting from all the transmitters. Each transmitter or receiver can choose its own position among the possible positions. Transmitters and receivers are working as players in a game to maximize its own contribution to the detection region with the errors below an acceptable criterion.

A dynamic game $G$ can be given by

$$G = \left[P, \{S_k\}_{k=1}^{M+N}, \{u_k\}_{k=1}^{M=N}\right], \tag{1}$$

where $P = \{p_1, p_2, \cdots, p_{M+N}\}$ denotes the players concerned in the game and their corresponding sequences, $S_k$ denotes the strategy space of the $k$th player and $u_k$ denotes the corresponding utility.

A receiver or a transmitter in the multistatic radar corresponds to a player in the game. The possible positions for the $k$th player are the strategy space and the performance improvement based on the CRLB of location accuracy is the utility.

Finite repeated game $G(Q)$ denotes repeating the game $G$ for $Q$ times where $Q$ is a finite number, and $G$ is the stage game in $G(Q)$. The sequence of players has been defined previously, and the $k$th player only knows the previous strategy in the game.

## 3. CRLB and available volume

The multistatic radar is assumed to be the system where only the bistatic range measurements are used for target localization. The geometric relationship between the receivers, transmitters and the target is given by

$$\begin{aligned} R_{m,n} = &\sqrt{(z - z_{Tm})^2 + (y - y_{Tm})^2 + (x - x_{Tm})^2} \\ &+ \sqrt{(z - z_{Rn})^2 + (y - y_{Rn})^2 + (x - x_{Rn})^2}, \end{aligned} \tag{2}$$

where $m = 1, 2, \cdots M$ and $n = 1, 2, \cdots N$. ($x_{Rn}$, $y_{Rn}$, $z_{Rn}$), ($x_{Tm}$, $y_{Tm}$, $z_{Tm}$) and ($x$, $y$, $z$) are the positions of the $n$th receiver, the $m$th transmitter and the target, respectively. The bistatic range estimation $R_{m,\,n}$ is assumed to be affected by the zero-mean random Gaussian error and the standard deviation is $\sigma_{R_{m,n}}$.

The CRLB with detection probability less than 1 is proposed in [2], and it is able to yield a realistic evaluation of the achievable location accuracy that fully includes the influences of the sensor placement and the SNR. The element $j$ and $h$ of the modified fisher information matrix (MFIM) [2] is given by

$$[\mathbf{MFIM}]_{j,h} = \sum_{k=1}^{K} P_k \sum_{m=1}^{M} \sum_{n=1}^{N} d_{m,n}^{(k)} \left(\frac{1}{\sigma_{R_{m,n}}^2} \frac{\partial R_{m,n}}{\partial \Theta_j} \frac{\partial R_{m,n}}{\partial \Theta_h}\right), \tag{3}$$

where

$$P_k = \prod_{m=1}^{M} \prod_{n=1}^{N} \left[d_{m,n}^{(k)} p_{d_{m,n}} + \left(1 - d_{m,n}^{(k)}\right)\left(1 - p_{d_{m,n}}\right)\right], \tag{4}$$

$$\sigma_{R_{m,n}} = \frac{c}{B\sqrt{2SNR_{m,n}}}. \tag{5}$$

The meaning of the parameters in (3)–(5) are presented as follows. $\Theta = [x, y, z]$ is the estimation parameter. $d_{m,n}^{(k)}$ is the binary variable and it is proposed to describe whether the target is detected by the bistatic pair consisting of the $m$th transmitter and the $n$th receiver. If $d_{m,n}^{(k)} = 1$, the target is detected, but if $d_{m,n}^{(k)} = 0$, the target is missing. $p_{d_{m,n}}$ is the detection probability of the target by the bistatic pair consisting of the $m$th transmitter and the $n$th receiver. Therefore, $MN$ binary variables generate $K = 2^{MN}$ possible determinations [2] and $P_k$ is the probability of the $k$th determination. $B$ is the bandwidth. $SNR_{m,\,n}$ is the SNR corresponding to bistatic pair consisting of the $m$th transmitter and $n$th receiver. $SNR_{m,\,n}$ is evaluated as

$$SNR_{m,n} = \frac{P_{t_m} G_{t_m} G_{r_n} \sigma_{m,n} \lambda T_{\text{int}}}{(4\pi)^3 \mathrm{k} T_s R_m^2 R_n^2 N_F}, \tag{6}$$

where $P_{t_m} G_{t_m}$ is the equivalent isotropically radiated power, $G_{r_n}$ is antenna gain, $\sigma_{m,\,n}$ is the target radar cross section, $\lambda$ is the wavelength, $T_{\text{int}}$ is the integration time, k is the Boltzmann constant, $T_s$ is the temperature and $N_F$ is equivalent noise figure.

The CRLB of location accuracy in space is given by

$$\sigma = \sqrt{\sum_{i=1}^{3} [\mathbf{MFIM}^{-1}]_{(i,i)}}, \tag{7}$$

where ($i$, $i$) means the $i$th row and $i$th column of the matrix.

The performance of location accuracy is evaluated by a fixed target position in [1,10,19]. Actually, further attentions are paid to the performance distribution in a surveillance area. The CRLB distribution is used in [2,3] to demonstrate the performance of location accuracy. As shown in [2,3], the shapes of CRLB distribution are not regular, and they are complex. It is worth noting that the mentioned distributions are calculated in a fixed height in [2,3]. Obviously, the target may locate anywhere in the surveillance area, so the fixed height is not appropriate. In order to evaluate the performance of location accuracy precisely, the surveillance area should be considered as a spatial region. The spatial region $S_a$ can be seen as a spatial point set. The set with CRLB of location accuracy below $\sigma_a$ is a subset of $S_a$ and is symbolized as $\Omega$, where $\sigma_a$ is the acceptable criterion of the CRLB. At this moment, $\Omega$ is an infinite point set and $\chi(\Omega)$ is the set cardinality. Different sensor placements correspond to different set cardinalities. The set cardinality can be seen as the performance metric of the layout. For example, $\chi(\Omega_1)$ and $\chi(\Omega_2)$ are set cardinalities in two different layout, respectively. If $\chi(\Omega_1) > \chi(\Omega_2)$, the layout corresponding to $\chi(\Omega_1)$ is better than the layout corresponding to $\chi(\Omega_2)$. However, the cardinality of an infinite set is difficult to calculate. For the sake of calculating conveniently, the surveillance area is divided into spatial regions in equal spacing. If the spacing is small enough, the CRLB of center position can be the representative of the CRLB of each divided spatial region. At this moment, $\Omega$ is a finite point set and its cardinality is easy to calculate. As known, the cardinality of finite set is the number of the element in the set.

In order to facilitate the understanding and associate with the physical properties of radar system, the spatial volume with the CRLB of location accuracy below the acceptable criterion $\sigma_a$ in the surveillance area is named as available volume. The spatial volume of each divided spatial region is assumed to be $w$ and $w = L \times W \times H$, where $L$, $W$ and $H$ are the length, width and height of the discrete region, respectively. Therefore, the available volume represents the amount of special coverage where the radar can detect target in an acceptable error. The available volume is a statistical property of the CRLB distribution and is given by $\chi(\Omega) \times$ w. The bigger the available volume is, the better the layout of multistatic radar is.

The set of all the players and the corresponding layout in the $q$th stage are given by

$$P(q) = \{p_1(q), p_2(q), \cdots, p_{M+N}(q)\}, \tag{8}$$

where $p_l(q)(l = 1, 2, \cdots, M + N)$ is the position of the $l$th player in the $q$th stage.