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Short communication

Alternating direction method of multipliers for radar waveform design in spectrally crowded environments

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ABSTRACT

This paper addresses the radar waveform design problem in spectrally crowded environments. The aim is to maximize the output signal-to-interference-plus-noise ratio (SINR) of the waveform under spectral and similarity constraints. The existing algorithm proposes to tackle such a problem via semidefinite relaxation (SDR) and rank-one decomposition, resulting in high complexity and limited applications. Motivated by the decomposability and superior convergence properties of alternating direction method of multipliers (ADMM), we propose a novel algorithm to tackle the waveform optimization problem. Since simpler subproblems are involved at each iteration and they can be tackled efficiently, the proposed algorithm has a much lower computational complexity than the existing algorithm. Numerical examples demonstrate the effectiveness of the proposed algorithm.

1. Introduction

The radio frequency (RF) electromagnetic spectrum is widely used in diverse applications including radar, communications, and navigation. In order to achieve improved target detection and parameter estimation performance, modern radar systems require large bandwidths. Meanwhile, the high-rate mobile broadband services also call for increased amount of bandwidth. Since the RF spectrum is a limited resource, the growing demand for more access to the spectrum by radar and communication systems leads to serious spectrum congestion problems. As a result, current radar systems face significant challenges to operate properly in spectrally crowded environments [1].

Several approaches have been developed to enhance the performance of radar systems in spectrally crowded environments. One possible means is to adapt the probing waveforms intelligently such that the adapted waveforms are compatible with the electromagnetic environments [2–14]. In particular, the optimization of waveforms under spectral constraints has received considerable interest in recent years. In [2], the authors proposed an algorithm to devise sparse frequency waveforms and receive filters. The devised waveforms could evenly distribute its spectral energy into the allowed bands and form wide notches in the stopbands. In [3], the authors proposed an iterative algorithm, called SHAPE, to devise waveforms with arbitrary spectral shapes. They also proposed a cyclic method to synthesize waveforms with both stopband and correlation constraints [4,5]. Different from the aforementioned works, the authors in [6–11] considered the waveform design problem via maximizing the output signal-to-interference-plus-noise-ratio (SINR), under the spectral and the similarity constraints on the waveforms. Typically, the associated optimization problems are non-convex. Nevertheless, it was shown in [6] that, when only one spectral compatibility constraint was enforced, the corresponding waveform design problem was a hidden-convex problem, which could be tackled based on semidefinite relaxation (SDR) and rank-one decomposition. However, the involved computational complexity is very high, and might be prohibitive for real-time waveform designs. Thus, computationally more efficient methods need to be developed.

In this paper, we revisit the waveform design problem in [6] and propose alternating direction method of multipliers (ADMM) to tackle the optimization problem. The decomposability of ADMM enables the proposed approach to only involve several quadratic programming (QP) subproblems during the iterations. More importantly, we can find efficient solutions to them. Owing to the simplicity of the proposed solution at every iteration and the superior convergence properties of ADMM, the proposed algorithm has a significantly lower computational complexity than the SDR-based approach.

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2. Problem formulation

Let $\mathbf{s} \in \mathbb{C}^{N\times 1}$ denote the baseband discrete-time probing waveform of a radar system, with N representing the code length. Consider the following signal model:

$$\mathbf{y} = \gamma_T \mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ stands for the received signal of the cell under test (CUT), γ_T is the target amplitude, and $\mathbf{n}$ denotes the (signal-independent) interference. It is well known that the target detection performance in Gaussian interference is closely related to the output SINR, and maximizing SINR leads to the largest probability of detection. Assume that $\mathbf{n}$ can be modeled as a complex-valued circularly symmetric Gaussian random vector, with zero-mean and covariance matrix $\mathbf{M}$. Then the output SINR can be defined as

$$\text{SINR} = |\gamma_T|^2 \mathbf{s}^H \mathbf{R} \mathbf{s}, \tag{2}$$

where $\mathbf{R} = \mathbf{M}^{-1}$ and $(\cdot)^H$ denotes conjugate transponse. Thus, we can seek to devise waveforms that maximize the output SINR to achieve the optimal detection performance.

Next we consider some practical constraints on the waveform of a radar system operating in a spectrally crowded environment. First, without loss of generality, we impose an energy constraint on the transmitted waveform, i.e., $\mathbf{s}^H\mathbf{s} = 1$; in addition, we enforce a similarity constraint on the waveform, i.e., $\|\mathbf{s} - \mathbf{s}_0\|^2 \le \varepsilon$, where $\mathbf{s}_0$ is the reference waveform, ε is a user-specified parameter which rules the similarity region, and $\|\cdot\|$ stands for the Euclidean norm. As shown in [15], enforcing a similarity constraint on the waveform controls the shape of its ambiguity function, partially circumventing the drawbacks including significant modulus variation, poor range resolution, and/or high peak side lobe levels. Finally, to ensure spectral compatibility with the surrounding licensed radiators, we have to shape the spectrum of the radar waveform. To this end, we impose a spectral constraint on the waveform:

$$\mathbf{s}^H \mathbf{R}_I \mathbf{s} \le E_I, \tag{3}$$

where E_I denotes the maximum allowed interference that can be tolerated by the licensed radiators, $\mathbf{R}_I = \sum_{k=1}^K w_k \mathbf{R}_I^k$, K represents the number of licensed radiators, w_k is the coefficient associated with the kth radiator,

$$\mathbf{R}_I^k(m,l) = \begin{cases} f_2^k - f_1^k, & m = l \\ \dfrac{e^{j2\pi f_2^k(m-l)} - e^{j2\pi f_1^k(m-l)}}{j2\pi(m-l)}, & m \ne l \end{cases}$$

f_1^k and f_2^k denote the lower and upper normalized working frequencies of the kth radiator, respectively.

Summing up the above results, we formulate the waveform optimization problem in a spectrally crowded environment as follows:

$$\max_{\mathbf{s}}\ \mathbf{s}^H\mathbf{R}\mathbf{s},\ \text{ s.t. } \mathbf{s}^H\mathbf{s} = 1,\ \|\mathbf{s}-\mathbf{s}_0\|^2 \le \varepsilon,\ \mathbf{s}^H\mathbf{R}_I\mathbf{s} \le E_I. \tag{4}$$

The optimization problem in (4) is a quadratically constrained quadratic programming (QCQP) problem. Since the objective function is convex with respect to (w.r.t.) $\mathbf{s}$ and the energy constraint is non-convex, the problem in (4) is difficult to solve. In [6], resorting to the framework of SDR and exploiting the hidden convexity of the problem, the authors showed that the optimal solution of (4) can be obtained within polynomial time. Particularly, the proposed algorithm therein involved solving a semidefinite programming (SDP) problem (with a complexity of $O(N^{4.5})\log(\zeta)$, where ζ is the solution accuracy [16]), and a rank-one decomposition procedure synthesizing the optimal waveform from the solution of the SDP (with a complexity of $O(N^3)$ [17]). Thus, the computational complexity of the proposed algorithm is quite high, which limits its applications.

3. Algorithm design

Motivated by the recent success of ADMM in large-scale optimizations [18–20], we propose a new algorithm to solve the optimization problem in (4). Before presenting the proposed algorithm, we first rewrite the problem in (4) as

$$\min_{\mathbf{s}}\ \mathbf{s}^H\mathbf{Q}\mathbf{s},\ \text{ s.t. } \mathbf{s}^H\mathbf{s} = 1,\ \|\mathbf{s}-\mathbf{s}_0\|^2 \le \varepsilon,\ \mathbf{s}^H\mathbf{R}_I\mathbf{s} \le E_I, \tag{5}$$

where $\mathbf{Q} = \mu\mathbf{I} - \mathbf{R}$ and μ is a positive constant to ensure $\mathbf{Q} \succeq \mathbf{0}$.

Considering that ADMM methods focus on real-valued problems, we define $\breve{\mathbf{s}} = [\Re(\mathbf{s}^T)\ \Im(\mathbf{s}^T)]^T$, $\breve{\mathbf{s}}_0 = [\Re(\mathbf{s}_0^T)\ \Im(\mathbf{s}_0^T)]^T$,

$$\breve{\mathbf{Q}} = \begin{bmatrix} \Re(\mathbf{Q}) & -\Im(\mathbf{Q}) \\ \Im(\mathbf{Q}) & \Re(\mathbf{Q}) \end{bmatrix}, \breve{\mathbf{R}}_I = \begin{bmatrix} \Re(\mathbf{R}_I) & -\Im(\mathbf{R}_I) \\ \Im(\mathbf{R}_I) & \Re(\mathbf{R}_I) \end{bmatrix},$$

where $(\cdot)^T$ stands for transpose, $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts, respectively. Then we can obtain $\mathbf{s}^H\mathbf{Q}\mathbf{s} = \breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}}$, $\mathbf{s}^H\mathbf{s} = \breve{\mathbf{s}}^T\breve{\mathbf{s}}$, $\mathbf{s}^H\mathbf{R}_I\mathbf{s} = \breve{\mathbf{s}}^T\breve{\mathbf{R}}_I\breve{\mathbf{s}}$, and $\|\mathbf{s}-\mathbf{s}_0\|^2 = \|\breve{\mathbf{s}}-\breve{\mathbf{s}}_0\|^2$.

Hence, we can reformulate (5) as the following real-valued optimization problem:

$$\min_{\breve{\mathbf{s}}}\ \ \breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}},\ \text{ s.t. } \breve{\mathbf{s}}^T\breve{\mathbf{s}} = 1,\ \|\breve{\mathbf{s}}-\breve{\mathbf{s}}_0\|^2 \le \varepsilon,\ \breve{\mathbf{s}}^T\breve{\mathbf{R}}_I\breve{\mathbf{s}} \le E_I. \tag{6}$$

To apply ADMM to the problem in (6), we use the variable splitting trick and introduce an auxiliary variable $\breve{\mathbf{z}}$:

$$\min_{\breve{\mathbf{s}},\breve{\mathbf{z}}}\ \ \breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}},\ \text{ s.t. } \breve{\mathbf{s}}^T\breve{\mathbf{s}} = 1,\ \|\breve{\mathbf{s}}-\breve{\mathbf{s}}_0\|^2 \le \varepsilon,\ \breve{\mathbf{z}}^T\breve{\mathbf{R}}_I\breve{\mathbf{z}} \le E_I,\ \breve{\mathbf{s}} = \breve{\mathbf{z}}. \tag{7}$$

The associated augmented Lagrangian is given by

$$L_\rho(\breve{\mathbf{s}},\breve{\mathbf{z}},\boldsymbol{\lambda}) = \breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}} + \boldsymbol{\lambda}^T(\breve{\mathbf{s}}-\breve{\mathbf{z}}) + \rho/2\|\breve{\mathbf{s}}-\breve{\mathbf{z}}\|^2, \tag{8}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier and ρ is the penalty parameter.

The proposed algorithm via ADMM consists of the following iterations:

$$\breve{\mathbf{s}}^{(k+1)} = \arg\min_{\breve{\mathbf{s}}} L_\rho(\breve{\mathbf{s}},\breve{\mathbf{z}}^{(k)},\boldsymbol{\lambda}^{(k)}), \tag{9}$$

$$\breve{\mathbf{z}}^{(k+1)} = \arg\min_{\breve{\mathbf{z}}} L_\rho(\breve{\mathbf{s}}^{(k+1)},\breve{\mathbf{z}},\boldsymbol{\lambda}^{(k)}), \tag{10}$$

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \rho(\breve{\mathbf{s}}^{(k+1)} - \breve{\mathbf{z}}^{(k+1)}). \tag{11}$$

Next we present the solutions for the problems in (9) and (10), respectively. For notational simplicity, we omit the variable superscripts in the following subproblems.

- **Update of $\breve{\mathbf{s}}^{(k+1)}$:**

$$\breve{\mathbf{s}}^{(k+1)} = \arg\min_{\breve{\mathbf{s}}}\ \{\breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}} + \boldsymbol{\lambda}^T(\breve{\mathbf{s}}-\breve{\mathbf{z}}) + \rho/2\|\breve{\mathbf{s}}-\breve{\mathbf{z}}\|^2\}$$
$$\text{s.t. } \breve{\mathbf{s}}^T\breve{\mathbf{s}} = 1,\ \|\breve{\mathbf{s}}-\breve{\mathbf{s}}_0\|^2 \le \varepsilon. \tag{12}$$

Note that the objective function of (12) can be written as

$$\breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}} + \boldsymbol{\lambda}^T(\breve{\mathbf{s}}-\breve{\mathbf{z}}) + \rho/2\|\breve{\mathbf{s}}-\breve{\mathbf{z}}\|^2$$
$$= \breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}} + (\boldsymbol{\lambda} - \rho\breve{\mathbf{z}})^T\breve{\mathbf{s}} + [\rho/2(1+\|\breve{\mathbf{z}}\|^2) - \boldsymbol{\lambda}^T\breve{\mathbf{z}}].$$

Thus the update equation of $\breve{\mathbf{s}}^{(k+1)}$ can be obtained by solving

$$\min_{\breve{\mathbf{s}}}\ \breve{\mathbf{s}}^T\breve{\mathbf{Q}}\breve{\mathbf{s}} + (\boldsymbol{\lambda} - \rho\breve{\mathbf{z}})^T\breve{\mathbf{s}},\ \text{ s.t. } \breve{\mathbf{s}}^T\breve{\mathbf{s}} = 1,\ \|\breve{\mathbf{s}}-\breve{\mathbf{s}}_0\|^2 \le \varepsilon. \tag{13}$$

One can solve the optimization problem in (13) via the Lagrange multipliers method. However, it has a high computational complexity. Alternatively, we apply majorization-minimization (MM) technique [21] to tackle it efficiently. To this end, we notice that $\breve{\mathbf{Q}} = \mu\mathbf{I} - \breve{\mathbf{R}} \preceq \breve{\mathbf{M}} = (\mu - \lambda_{\min}(\breve{\mathbf{R}}))\mathbf{I}$ (here $\breve{\mathbf{R}}$ is the real-valued version of $\mathbf{R}$ defined like $\breve{\mathbf{Q}}$ and $\lambda_{\min}(\breve{\mathbf{R}})$ is its smallest eigenvalue). Then $(\breve{\mathbf{s}} - \breve{\mathbf{s}}^{(k,n)})^T(\breve{\mathbf{Q}} - \breve{\mathbf{M}})(\breve{\mathbf{s}} - \breve{\mathbf{s}}^{(k,n)}) \le 0$. As a result, we can find a surrogate function of the objective in (13),