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Short communication

Greedy Pursuits Assisted Basis Pursuit for reconstruction of joint-sparse signals

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ABSTRACT

Distributed Compressive Sensing (DCS) is an extension of compressive sensing from single measurement vector problem to Multiple Measurement Vectors (MMV) problem. In DCS, several reconstruction algorithms have been proposed to reconstruct the joint-sparse signal ensemble. However, most of them are designed for signal ensemble sharing common support. Since the assumption of common sparsity pattern is very restrictive, we are more interested in signal ensemble containing both common and innovation components. With a goal of proposing an MMV-type algorithm that is robust to outliers (absence of common sparsity pattern), we propose Greedy Pursuits Assisted Basis Pursuit for Multiple Measurement Vectors (GPABP-MMV). It employs modified basis pursuit and MMV versions of multiple greedy pursuits. We also formulate the exact reconstruction conditions and the reconstruction error bound for GPABP-MMV. GPABP-MMV is suitable for a variety of applications including time-sequence reconstruction of video frames, reconstruction of ECG signals, etc.

1. Introduction

Compressive Sensing (CS) [1] ensures the reconstruction of a sparse signal $x \in \mathbb{R}^n$ from $m \ll n$ linear incoherent measurements of the form $y = \Phi x \in \mathbb{R}^m$ where $\Phi \in \mathbb{R}^{m\times n}$ is a known sensing matrix. CS reconstruction algorithms can be broadly classified as convex relaxation methods (such as Basis Pursuit (BP) [2]) and Greedy Pursuit (GP) algorithms (such as Orthogonal Matching Pursuit (OMP) [3] and Subspace Pursuit (SP) [4]). For multiple signals, Single Measurement Vector (SMV) reconstruction problem can be extended to Multiple Measurement Vectors (MMV) problem [5]. The joint-sparse signal ensemble (otherwise known as correlated sparse signal ensemble) can be broadly classified as innovative joint-sparse and common joint-sparse. Let J denote the number of signals in the joint-sparse signal ensemble, $x_j \in \mathbb{R}^n$ denote the signal j where $j \in \{1, 2, \ldots, J\}$, and K denote the number of non-zero elements in each x_j. In the innovative joint-sparse signal ensemble (otherwise known as Joint-Sparsity Model (JSM) – 1), each x_j consists of two components: a common sparse component that is present in all the signals, and a sparse innovation component that is unique to it [6]. In other words, x_j can be split as $x_j = z + z_j$ where z is the same for all of the x_j and z_j is the unique portion of x_j. Let K_c denote the number of non-zero elements in z. In the common joint-sparse signal ensemble (otherwise known as JSM-2), all the signals share a common sparsity pattern. In other words, each x_j is supported only on the same set of K non-zero locations but with different non-zero coefficients [6].

Each signal in the ensemble, x_j, is independently sensed using Φ such that $y_j = \Phi x_j \in \mathbb{R}^m$, and then the resulting measurements are transmitted. The data matrix for an ensemble of J signals (in the case of noiseless measurements) is given by,

$$\mathbf{Y} = \Phi \mathbf{X} \ \in \mathbb{R}^{m\times J} \tag{1}$$

where $\mathbf{Y} = [y_1, y_2, \ldots., y_J]$ and $\mathbf{X} = [x_1, x_2, \ldots., x_J] \in \mathbb{R}^{n\times J}$.

1.1. Motivation and relation to prior work

For joint-sparse signals sharing a common sparsity pattern (i.e. common joint-sparse signals), several algorithms have been developed to reconstruct $\mathbf{X}$ from $\mathbf{Y}$, utilizing the common sparsity condition [7–12]. However, in most cases this knowledge does not exist and the assumption of common sparsity pattern for 100% of the data becomes far from ideal. Therefore, we are interested in innovative joint-sparse signals. In [13], a SMV-type solution was proposed for reconstructing innovative joint-sparse signals. It solves a single linear program involving concatenated measurement vectors and concatenated measurement matrices. Due to concatenations, dimension of the signal to be reconstructed

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increased from n to nJ, and therefore, the complexity of the algorithm is high. An MMV-type recovery method for innovative joint-sparse signals, the Texas Hold 'Em algorithm [14], separated the recovery of the common and innovation components into two stages. The measurements used to recover the common component are obtained by either averaging or concatenating measurements from all the sensors in the network. Furthermore, each innovation component is recovered locally at the corresponding sensor. The computational complexity of the Texas Hold 'Em algorithm is linear in J. However, due to the fact that the recovery of innovation components is based on the average or concatenated measurements, its achievable reconstruction accuracy is limited. To recover two correlated signals with partially disjoint supports, a modified Orthogonal Matching Pursuit (OMP) algorithm called Probability weighted OMP (P-OMP) was proposed [15]. However, it has a drawback: it requires prior probabilistic information on the signals supports. Such information may not be available in many practical applications and therefore, P-OMP cannot be applied in those applications. In [16], a Robust Multiple Sparse Bayesian Learning (R-MSBL) algorithm was proposed which captured the support set of the majority of data vectors. Though R-MSBL, unlike P-OMP, does not require any probabilistic information, the time it takes for reconstruction is a concern. This concern arises mainly due to the presence of innovation components.

Certain optimization-based algorithms attempted to recover $\mathbf{X}$ by mixed-norm minimization [17]. Mixed-norms exploit both sparsity and structure in the signal ensemble. Several mixed-norm based optimization algorithms have been designed to handle structured sparsity [18,19]. For data ensemble with simultaneous low-rank and joint-sparse structure, a CS recovery approach jointly regularizing the solutions with their nuclear norm and the $\ell_{2,1}$-norm is proposed [9]. However, these mixed-norm based algorithms are not robust to innovation components present in x_j. Therefore, we seek an MMV-type algorithm that is robust to innovation components.

1.2. Contributions

In this article, we propose Greedy Pursuits Assisted Basis Pursuit for Multiple Measurement Vectors (GPABP-MMV), and formulate its exact reconstruction conditions and the reconstruction error bound. Through extensive simulations, we show that the GPABP-MMV is robust to outliers and it outperforms state-of-the-art MMV-type algorithms (such as R-MSBL and Texas Hold 'Em) in terms of reconstruction accuracy. GPABP-MMV is suitable for applications wherein the signal ensemble follows innovative joint-sparse formulation with K_c significantly less than K. The rest of this paper is organized as follows. In Section 2, we briefly discuss Greedy Pursuits Assisted Basis Pursuit (GPABP) [20]. In Section 3, we propose and analyze GPABP-MMV. In Section 4, we present the simulation results comparing the performance of GPABP-MMV to that of the R-MSBL, Texas Hold 'Em, etc. Section 5 concludes the paper.

2. Greedy Pursuits Assisted Basis Pursuit

Consider the SMV reconstruction problem (i.e. reconstruction of x from y). The actual support of x, $T \subset \{1, 2, \dots, n\}$, is defined as the set of indices i where $x(i)$ is non-zero. The partial support, $\Lambda \subset \{1, 2, \dots, n\}$, is defined as the set of indices i where $x(i)$ is estimated to be non-zero. In some CS reconstruction problems, the partial support is known (termed as Partially Known Support (PKS)) and it is used to run Modified Basis Pursuit (Mod-BP) [21]. In [20], it was shown that, if the partial support is not known, it can be derived using multiple GPs and then Mod-BP can be applied. The GPABP algorithm [20], given as algorithm 1, employs multiple GPs to form Λ. Let L denote the number of GPs involved in the formation of Λ. In fact, the L GPs are L different GPs. For example, if $L = 2$, OMP is chosen for GP_1 and SP is chosen for GP_2. Therefore, L GPs can give up to L different solutions of the form

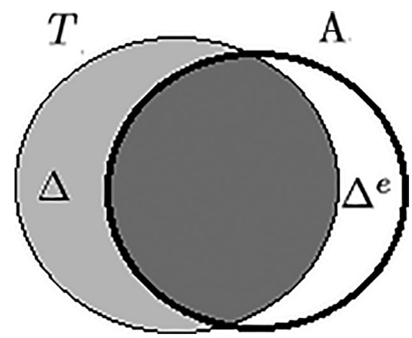

Fig. 1. Schematic of actual support (T) and partial support (Λ).

$$\hat{T}_i = \text{GP}_i(\Phi, y, K) \quad \forall i \in \{1, 2, \dots, L\}$$

where GP_i stands for ith greedy pursuit and $\hat{T}_i$ is the support set estimated by GP_i. Then, the partial support is formed as follows:

$$\Lambda := \bigcap_{i=1}^{L} \hat{T}_i.$$

The need for L different GPs to derive Λ is as follows. Reconstruction performance of a GP varies and depends on the nature of the sparse signal [22,23]. For example, OMP performs better than SP for some types of signals and SP performs better than OMP for some other type of signals. If the statistical distribution of the non-zero values of the signal is known a priori, the best recovery algorithm (for that type of signal) can be applied to derive the partial support. But in many practical scenarios, this prior knowledge is not available and hence, a fusion based approach (as in [24]) that utilizes several GPs is used to obtain the partial support.

Upon obtaining Λ, x is reconstructed from y using Mod-BP (termed Mod-CS in [21]). The Mod-BP problem (in the case of noiseless measurements) is formulated as

$$\hat{x} = \arg\min_{\tilde{x}} \|\tilde{x}_{\Lambda^c}\|_1 \text{ s.t. } \Phi\tilde{x} = y \tag{2}$$

where Λ^c is the set compliment of Λ, $\tilde{x}_{\Lambda^c}$ is the subset of $\tilde{x}$ formed by extracting the entries of $\tilde{x}$ corresponding to the indices in Λ^c, $\|.\|_1$ stands for the vector ℓ_1-norm and $\hat{x}$ is the reconstructed signal. Note that, in the case of noisy measurements, the constraint of the Mod-BP problem will be $\|\Phi\tilde{x} - y\|_2 \le \|w\|_2$ where $w \in \mathbb{R}^m$ is an additive noise such that $y = \Phi x + w \in \mathbb{R}^m$.

Lemma 1 recalls the exact reconstruction conditions for GPABP (in the case of noiseless measurements) as given in [21] and Lemma 2 recalls the reconstruction error bound for GPABP (in the case of noisy measurements) as given in [25]. The actual support of x, T, is split as $T = (\Lambda \cup \Delta)\setminus\Delta^e$ where $\Delta := T\setminus\Lambda$ is the part of the support missing in Λ and $\Delta^e := \Lambda\setminus T$ is the set of wrong locations in Λ. A schematic describing T, Λ, Δ, and Δ^e is given in Fig. 1. Consider a typical reconstruction example where $n = 50$, $T = \{5, 10, 15, 20, 25, 30, 35, 40\}$ and $\Lambda = \{5, 10, 15, 20, 25, 30, 45\}$. Then, $\Delta = \{35, 40\}$ and $\Delta^e = \{45\}$. The signal sparsity (i.e. the number of non-zero elements in x) is related to the signal support as $K = |T|$ where $|.|$ stands for the cardinality of a set. The Restricted Isometry Constant (RIC), δ_S, for $\Phi \in \mathbb{R}^{m \times n}$ is as defined in [2].

Lemma 1 (Theorem 1 in [21])**.** *Let $y = \Phi x \in \mathbb{R}^m$. If Λ obeys $T := (\Lambda \cup \Delta)\setminus\Delta^e$, then $\hat{x}$ is the unique minimizer of Mod-BP (in step 3 of Algorithm 1) if*

$$|\Delta| \le |\Lambda| \text{ and } \delta_{K+|\Delta|+|\Delta^e|} < \frac{1}{5}.$$