# A novel algebraic solution for moving target localization in multi-transmitter multi-receiver passive radar 

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#### Abstract

This paper investigates the problem of locating a moving target using a passive radar system with multiple transmitters and multiple receivers. The bistatic range and bistatic range rate between each transmitter and receiver are used as the measurements. A novel algebraic solution employing two-step weighted least squares ( 2 WLS ) minimizations is proposed. In the first step, the measurement equations are linearized by introducing multiple additional parameters and a WLS minimization is used to obtain a rough estimate; then in the second step, the known relation between the additional parameters and the target location parameters is utilized to refine the estimate. Theoretical accuracy analysis indicates that the proposed algorithm achieves the Cramer-Row lower bound, and Monte-Carlo simulations demonstrate that the proposed algorithm outperforms existing algorithms.


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## 1. Introduction

Passive radar, also referred to as passive coherent location and passive covert radar, uses third-party transmitters in the environment to detect and track targets [1]. In recent years, the use of passive radar for surveillance purposes has received renewed interest in both civilian and military fields due to a number of advantages such as covertness, wide coverage, low cost of operation and maintenance, operation without a frequency clearance, and capabilities against stealth aircraft $[2,3]$.

A distinct feature of passive radar is the presence of two receiving channels, one for the reference signal arriving directly from the transmitter and the other for the signal via reflection from the target [4]. By conducting delay-Doppler cross-correlation between the reflected signal and the reference signal, the time delay (TD) and Doppler shift (DS) can be measured, which are then used to estimate the target position and velocity [5]. The TD is directly related to the bistatic range ( BR ), which is the sum of transmittertarget and target-receiver ranges [6], and the DS is related to the bistatic range rate (BRR). However, unlike the range difference (RD) and range rate difference (RRD) based localization, which has been extensively studied [7-14], BR-and-BRR-based localization is more challenging and relatively little work is available in open literature.

Recently, some efforts have been devoted to solving this challenging problem. Algebraic solutions have been of interest to re-

[^0]searchers for their computational efficiency and independence on initial guess. Borrowing the idea of the well-known two-step weighted least squares (2WLS) method by Chan and Ho [7], Du and Wei [15] explored an algebraic solution for moving target localization with a noncoherent distributed MIMO radar using delay and Doppler shift measurements. To reduce solving difficulty, they firstly divide the measurements into several groups based on different transmitters or receivers, then employ two WLS estimators for each group to independently produce an estimate of target position and velocity, and finally, these results from different groups are combined to form a composite estimate. For convenience, we will refer to the algorithm proposed in [15] as 'Group2 WLS ' since its distinct feature is to form virtual groups. However, Group-2WLS suffers from threshold effect, and furthermore dividing the measurements into groups and optimizing each group separately does not guarantee a global optimal solution in a WLS sense if the measurement noises are correlated. This has motivated researchers to consider more efficient designs. More recently, Yang and Chun [16] developed an improved algebraic solution, which is also based on the 2WLS idea, but in contrast to the Group-2WLS, it linearizes the measurement equations by selecting one of the receivers or transmitters as the reference and converting the $B R$ and BRR measurements to the RD and RRD ones. For short, we refer to this algorithm as 'Difference-2WLS'. Difference-2WLS algorithm does not need grouping and combining, but converting the $B R$ and BRR measurements to the RD and RRD ones leads to a loss in the number of equations, which lowers the localization accuracy. Hence, there is yet a need for developing an algebraic without grouping or a loss in the number of equations.


Fig. 1. An example of system geometry in a 2-D situation.

Motivated by these facts, we derive in this paper a new algebraic solution for moving target localization in multi-transmitter multi-receiver passive radar systems using BR and BRR measurements. The proposed solution is inspired by the 2WLS idea [7], but in contrast to recently proposed Group-2WLS and Difference2WLS, it does not need to group or convert the BR and BRR measurements to the RD and RRD ones. The proposed solution needs two WLS steps: In the first WLS step, the measurement equations are linearized by introducing multiple additional parameters and a WLS estimator is used to obtain a rough estimate; then in the second WLS step, the known relation between the additional parameters and the target location parameters is utilized to refine the estimate. The proposed solution gives higher accuracy than existing algorithms [15,16]. Moreover, theoretical accuracy analysis demonstrates that the proposed solution achieves the Cramer-Rao lower bound (CRLB).

The remainder of this paper is organized as follows. In the next section, we formulate the problem of moving target localization for multi-transmitter multi-receiver passive radar using BR and BRR measurements. The algebraic solution of the target position and velocity is derived in Section 3. Section 4 analyzes the theoretical accuracy of the proposed solution and compares it with the CRLB, with simulation results given in Section 5. Finally, some conclusions are drawn in Section 6.

## 2. Measurement model

We consider the problem of localizing a moving target in 2D plane, as illustrated in Fig. 1, using a passive radar system with $M$ noncoherent transmitters, which can be analog television signals, FM radio signals, cellular phone base stations, digital audio broadcasting, digital video broadcasting, satellites and so on [4], and $N$ receivers, which can be integrated multi-band antenna systems deployed on stationary platforms or moving vehicles [1]. The target position and velocity of the target, denoted by $\boldsymbol{u}=\left[\begin{array}{cc}x & y\end{array}\right]^{\mathrm{T}}$ and $\dot{\boldsymbol{u}}=\left[\begin{array}{cc}\dot{x} & \dot{y}\end{array}\right]^{\mathrm{T}}$, are to be determined. The position and velocity of the $m$ th transmitter are $\boldsymbol{s}_{m}^{t}=\left[\begin{array}{lll}x_{m}^{t} & y_{m}^{t}\end{array}\right]^{\mathrm{T}}$ and $\dot{\boldsymbol{s}}_{m}^{t}=$ [ $\left.\begin{array}{lll}\dot{x}_{m}^{t} & \dot{y}_{m}^{t}\end{array}\right]^{\mathrm{T}}$, and the position and velocity of the $n$th receiver are $\boldsymbol{s}_{n}^{r}=\left[\begin{array}{lll}x_{n}^{r} & y_{n}^{r}\end{array}\right]^{\mathrm{T}}$ and $\dot{\boldsymbol{s}}_{n}^{r}=\left[\begin{array}{lll}\dot{x}_{n}^{r} & \dot{y}_{n}^{r}\end{array}\right]^{\mathrm{T}}$, respectively.

The range and range-rate between the target and the $m$ th transmitter are, respectively, given by
$R_{m}^{t}=\left\|\boldsymbol{u}-\boldsymbol{s}_{m}^{t}\right\|$
$\dot{R}_{m}^{t}=\frac{\left(\boldsymbol{u}-\boldsymbol{s}_{m}^{t}\right)^{\mathrm{T}}\left(\dot{\boldsymbol{u}}-\dot{\boldsymbol{s}}_{m}^{t}\right)}{R_{m}^{t}}$

Similarly, the range and range-rate between the target and the $n$th receiver are, respectively, given by
$R_{n}^{r}=\left\|\boldsymbol{u}-\boldsymbol{s}_{n}^{r}\right\|$
$\dot{R}_{n}^{r}=\frac{\left(\boldsymbol{u}-\boldsymbol{s}_{n}^{r}\right)^{\mathrm{T}}\left(\dot{\boldsymbol{u}}-\dot{\boldsymbol{s}}_{n}^{r}\right)}{R_{n}^{r}}$
where $\left\|^{*}\right\|$ represents the 2-norm. According to the definition, the $B R$ and BRR corresponding to the $m$ th transmitter and $n$th receiver, are $r_{m, n}^{o}=R_{m}^{t}+R_{n}^{r}$ and $\dot{r}_{m, n}^{o}=\dot{R}_{m}^{t}+\dot{R}_{n}^{r}$, respectively. Considering the measurement noises in practice, the BR and BRR measurements converted from the TD and DS measurements, denoted by $r_{m, n}$ and $\dot{r}_{m, n}$ respectively, are modelled as
$r_{m, n}=r_{m, n}^{o}+\Delta r_{m, n}$
$\dot{r}_{m, n}=\dot{r}_{m, n}^{o}+\Delta \dot{r}_{m, n}$
where $\Delta r_{m, n}$ and $\Delta \dot{r}_{m, n}$ are additive, zero-mean Gaussian noises. Let $\quad \begin{array}{r}r\end{array}=\left[\begin{array}{llll}\boldsymbol{r}_{1}^{\mathrm{T}} & \boldsymbol{r}_{2}^{\mathrm{T}} & \ldots & \boldsymbol{r}_{M}^{\mathrm{T}}\end{array}\right]^{\mathrm{T}}$ with $\boldsymbol{r}_{m}=\left[\begin{array}{llll}r_{m, 1} & r_{m, 2} & \ldots & r_{m, N}\end{array}\right]^{\mathrm{T}}$, and $\quad \dot{\boldsymbol{r}}=\left[\begin{array}{llll}\dot{\boldsymbol{r}}_{1}^{\mathrm{T}} & \dot{\boldsymbol{r}}_{2}^{\mathrm{T}} & \ldots & \dot{\boldsymbol{r}}_{M}^{\mathrm{T}}\end{array}\right]^{\mathrm{T}}$ with $\dot{\boldsymbol{r}}_{m}=\left[\begin{array}{llll}\dot{r}_{m, 1} & \dot{r}_{m, 2} & \ldots & \dot{r}_{m, N}\end{array}\right]^{\mathrm{T}}$, be the vectors of noisy BR measurements and BRR measurements, $\quad \boldsymbol{r}^{o}=\left[\begin{array}{llll}\left(\boldsymbol{r}_{1}^{o}\right)^{\mathrm{T}} & \left(\boldsymbol{r}_{2}^{o}\right)^{\mathrm{T}} & \ldots & \left(\boldsymbol{r}_{M}^{o}\right)^{\mathrm{T}}\end{array}\right]^{\mathrm{T}} \quad$ with $\quad \boldsymbol{r}_{m}^{o}=$ $\left[\begin{array}{llll}r_{m, 1}^{o} & r_{m, 2}^{o} & \ldots & r_{m, N}^{o}\end{array}\right]^{\mathrm{T}}, \quad$ and $\quad \dot{\boldsymbol{r}}^{0}=\left[\begin{array}{llll}\left(\dot{\boldsymbol{r}}_{1}^{o}\right)^{\mathrm{T}} & \left(\dot{\boldsymbol{r}}_{2}^{o}\right)^{\mathrm{T}} & \ldots & \left(\dot{\boldsymbol{r}}_{M}^{o}\right)^{\mathrm{T}}\end{array}\right]^{\mathrm{T}}$ with $\dot{\boldsymbol{r}}_{m}^{o}=\left[\begin{array}{llll}\dot{r}_{m, 1}^{0} & \dot{r}_{m, 2}^{0} & \ldots & \dot{r}_{m, N}^{0}\end{array}\right]^{\mathrm{T}}$, be their true values. Stacking the BR and BRR measurements for the $M$ transmitters and $N$ receivers, yields respectively
$\boldsymbol{r}=\boldsymbol{r}^{0}+\Delta \boldsymbol{r}$
$\dot{\boldsymbol{r}}=\dot{\boldsymbol{r}}^{0}+\Delta \dot{\boldsymbol{r}}$
where $\quad \Delta \boldsymbol{r}=\left[\begin{array}{llll}\Delta \boldsymbol{r}_{1}^{\mathrm{T}} & \Delta \boldsymbol{r}_{2}^{\mathrm{T}} & \ldots & \Delta \boldsymbol{r}_{M}^{\mathrm{T}}\end{array}\right]^{\mathrm{T}} \quad$ with $\quad \Delta \boldsymbol{r}_{m}=$ $\left[\begin{array}{llll}\Delta r_{m, 1} & \Delta r_{m, 2} & \ldots & \Delta r_{m, N}\end{array}\right]^{\mathrm{T}}, \quad$ and $\quad \Delta \dot{\boldsymbol{r}}=\left[\begin{array}{llll}\Delta \dot{\boldsymbol{r}}_{1}^{\mathrm{T}} & \Delta \dot{\boldsymbol{r}}_{2}^{\mathrm{T}} & \ldots & \Delta \dot{\boldsymbol{r}}_{M}^{\mathrm{T}}\end{array}\right]^{\mathrm{T}}$ with $\Delta \dot{\boldsymbol{r}}_{m}=\left[\begin{array}{llll}\Delta \dot{r}_{m, 1} & \Delta \dot{r}_{m, 2} & \ldots & \Delta \dot{r}_{m, N}\end{array}\right]^{\mathrm{T}}$, are the vectors of BR and BRR measurement noises. Putting the two sets of measurements together, we obtain the total measurements vector $\left[\boldsymbol{r}^{\mathrm{T}} \dot{\boldsymbol{r}}^{\mathrm{T}}\right]^{\mathrm{T}}$. The corresponding measurement error vector is $\left[\Delta \boldsymbol{r}^{\mathrm{T}} \Delta \dot{\boldsymbol{r}}^{\mathrm{T}}\right]^{\mathrm{T}}$, which is zero-mean Gaussian with covariance matrix
$\mathrm{E}\left\{\left[\Delta \boldsymbol{r}^{\mathrm{T}} \quad \Delta \dot{\boldsymbol{r}}^{\mathrm{T}}\right]^{\mathrm{T}}\left[\Delta \boldsymbol{r}^{\mathrm{T}} \quad \Delta \dot{\boldsymbol{r}}^{\mathrm{T}}\right]\right\}=\boldsymbol{Q}$
where the covariance matrix $\mathbf{Q}$ is usually determined from the specific signal conditions [17]. Obviously, there exist MN BR measurements and $M N$ BRR measurements, with regard to the number of transmitters and receivers. Our task is to determine the target position and velocity from these noisy measurements.

## 3. Localization algorithm

### 3.1. First WLS step

Determining the target position and velocity from BR and BRR measurements obtained at a single time instant is not a trivial task, because the target location is nonlinearly related to the measurements. To overcome this difficulty, we first linearize them. By moving $R_{m}^{t}$ to the left side, we rearrange (5) as
$r_{m, n}-R_{m}^{t}=R_{n}^{r}+\Delta r_{m, n}$
Squaring both sides of (10) and using (1) and (3), we have

$$
\begin{align*}
r_{m, n}^{2} & -2 r_{m, n} R_{m}^{t}+\left(\boldsymbol{u}^{\mathrm{T}} \boldsymbol{u}-2\left(\boldsymbol{s}_{m}^{t}\right)^{\mathrm{T}} \boldsymbol{u}+\left(\boldsymbol{s}_{m}^{t}\right)^{\mathrm{T}} \boldsymbol{s}_{m}^{t}\right) \\
& =\left(\boldsymbol{u}^{\mathrm{T}} \boldsymbol{u}-2\left(\boldsymbol{s}_{n}^{r}\right)^{\mathrm{T}} \boldsymbol{u}+\left(\boldsymbol{s}_{n}^{r}\right)^{\mathrm{T}} \boldsymbol{s}_{n}^{r}\right)+2 R_{n}^{r} \Delta r_{m, n}+\left(\Delta r_{m, n}\right)^{2} \tag{11}
\end{align*}
$$

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