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A time varying filter approach for empirical mode decomposition

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ABSTRACT

A modified version of empirical mode decomposition (EMD) is presented to solve the mode mixing problem. The sifting process is completed using a time varying filter technique. In this paper, the local cut-off frequency is adaptively designed by fully facilitating the instantaneous amplitude and frequency information. Then nonuniform B-spline approximation is adopted as a time varying filter. In order to solve the intermittence problem, a cut-off frequency realignment algorithm is also introduced. Aimed at improving the performance under low sampling rates, a bandwidth criterion for intrinsic mode function (IMF) is proposed. The proposed method is fully adaptive and suitable for the analysis of linear and non-stationary signals. Compared with EMD, the proposed method is able to improve the frequency separation performance, as well as the stability under low sampling rates. Besides, the proposed method is robust against noise interference.

1. Introduction

For many type of signals, such as vibration signals, water waves and biological signals, it is often desirable to decompose the given signal into simple components (or mono-component signals) for further analysis. One of the most common examples is in the field of time-frequency analysis, where the Hilbert spectrum exhibits valuable performance for mono-component signals, but fails to provide meaningful information for multi-component signals [1]. Empirical mode decomposition (EMD) [2] is a method for decomposing complex, multi-component signals into several elementary oscillations, called intrinsic mode functions (IMFs). An IMF has to satisfy the following two conditions [2]: (1) the number of extrema and zero-crossings must either be equal or differ at most by one and (2) the local average of the upper and lower envelopes is zero. Although EMD has been proved remarkably effective in many applications [3–6], some problems still remain. One of these problems is that EMD fails to distinguish components whose frequencies lie within an octave [7] (we refer to this as the separation problem). Another problem is that EMD is vulnerable to intermittence such as noise [8] (we refer to this as the intermittence problem). As a consequence, the resulting IMFs may contain widely spread scales, known as the mode mixing problem. Since both the separation problem and the intermittence problem can cause mode mixing, a number of methods are proposed to address the two problems.

For the separation problem, it is reported that EMD can separate two components only when their frequency ratio is below a particular cut-off (about 0.65). In order to achieve a better separation performance, specific models, such as linear models [9,10] and AM-FM models [11–13], are studied. However, these specialized models are difficult to use for real-life signals. Since EMD involves the interpolation of local extrema, some methods are proposed by finding alternative points for interpolation [14,15], and achieving a separation performance better than 0.8. However, these methods are often complicated and difficult to interpret. Later, the focus of research was on more comprehensive ways, where analytic methods [16,17] are proposed. The analytic methods defined analytical expressions, and some achieved a separation performance better than 0.9.

For the intermittence problem, noise-assisted techniques such as ensemble EMD (EEMD) [18] and noise-assisted MEMD (N-A MEMD) [19,20] are popular methods, which are based on the filter bank structure of EMD [21]. With EEMD and N-A MEMD, the decomposed results were uniformly aligned according to the cut-off frequency bands. However, the noise-assisted methods have some disadvantages: (1) Parameters such as amplitude of noise and ensemble number are quite difficult to choose, making it no longer adaptive. (2) It still fails to separate modes whose frequencies lie within an octave. (3) If we regard EMD as a filtering

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process, its cut-off frequency is time-varying. On the contrary, the cut-off frequency of the noise-assisted methods is constant, which is not suitable for non-stationary signals. Besides the noise-assisted techniques, filtering based methods such as variational mode decomposition (VMD) [22] and empirical wavelet transform (EWT) [23] have been recently proposed, which are based on Wiener filtering and wavelet filter bank respectively. Although VMD and EWT have been proved to be robust to noise, these methods require an appropriate choice of parameters.

In this paper, we alleviate the mode mixing problem by the use of time varying filter (TVF). The cut-off frequency of a TVF is time varying, which is suitable for non-stationary signals. Our idea is quite simple: find out the local cut-off frequency and then perform time varying filtering. During the sifting process, the given signal is filtered and divided into two parts, i.e., the local higher frequency (LHF) component and the local lower frequency (LLF) one. By successively applying the TVF, the obtained LHF component is a local narrow-band signal, which has similar properties to the IMF.

Compared with existing methods, our proposed method has the following characteristics. Firstly, we fully address the separation problem [7] as well as the intermittence problem [8]. By contrast, many of the existing methods deal with the two problems separately, such as [9–13] and [18–20,23]. Secondly, our method adopts TVF in the sifting process. Thus, it is able to solve the mode mixing problem while simultaneously preserving the time varying feature, which is a very important aspect of EMD. By contrast, the filter cut-off frequencies of the filtering based methods, such as EEMD, N-A MEMD, VMD and EWT, are constant with respect to time, which is not suitable for non-stationary signals. Thirdly, the stopping criterion is improved, by considering signals under low sampling rates. We require the resulting IMF to be of a local narrow-band, and do not require the upper and lower envelopes to be symmetric. This is different from most of the existing methods. As a result, our method is able to achieve a robust performance under low sampling rates. Finally, the parameters in our method have clear physical meanings and are easy to be selected. By contrast, some existing methods such as EEMD, VMD and EWT, require many parameters, and the parameters are not easy to be selected adaptively. The proposed method is referred to as time varying filtering based EMD (TVF-EMD). As a supplement, the MATLAB source code for TVF-EMD is available from http://samd.site90.com/tvfemd.php.

The remainder of the paper is organized as follows. Section 2 briefly introduces the principle of EMD and the limitations of IMF. In Section 3 we show that B-spline approximation is a special form of TVF. In Section 4, we focus on the central topic of our work, i.e., how to estimate the filter cut-off frequency, which is capable of dealing with both the separation problem and the intermittence problem. In Section 5, based on the concepts outlined in Sections 3 and 4, we present the sifting process of our proposed method. In this section, we also discuss the stopping criterion and some properties of the proposed method. Section 6 provides experimental validation of our method along with performance comparisons. Finally in Section 7, we will offer a conclusion to our paper.

2. EMD, IMF and local narrow-band signal

2.1. Empirical mode decomposition

EMD decompose a given signal $x(t)$ into a finite set of mono-component like components (IMFs), plus a nonzero mean residual $r(t)$.

$$x(t) = \sum_{i=1}^{N} imf_i(t) + r(t) \tag{1}$$

where $imf_i(t)$ is the i-th IMF. To obtain each IMF, an iterative procedure called the sifting process is used. The sifting process of EMD is mainly carried out by two steps: (1) Estimate the "local mean". (2) Recursively subtract the local mean from the input signal until the resulting signal becomes an IMF. Given an input signal$x(t)$, let$h(t) = x(t)$, the sifting process of EMD can be summarized as Algorithm 1.

Due to the lack of an exact definition of the mono-component, IMF is used as an alternative. However, IMF does not guarantee that it has only one oscillatory mode. As we will discuss later, it cannot even guarantee that IMF is narrow-band. In EMD, the local mean is defined as the average of the upper and the lower envelopes, and the upper and lower envelopes are obtained by cubic spline interpolation. This process is difficult to express as an analytic formula. Besides, the obtained IMF may not be meaningful due to its limitations.

2.2. Limitations of IMF

In EMD, IMF is used to replace the mono-component signal based on the observations of the narrow band signal [2]. Recalling the second condition of IMF, the local average of the upper and lower envelope of an IMF should be zero. Although the second condition of IMF is to make the instantaneous frequency of an IMF meaningful, it has its limitations.

The first limitation is that the second condition of IMF is too rigid for stopping criterion. The local mean may not be zero everywhere, even the input signal is narrow-band. In order to make the stopping criterion easier to be realized, Huang [2] defined a stopping criterion called standard deviation (SD), by limiting the variation between two consecutive sifting results. Rilling [24] proposed a 3-threshold criterion and Damerval [25] proposed a criterion based on the number of iterations. None of the aforementioned criteria explore the physical information and therefore have no reasonable interpretation.

The second limitation is that the second condition of IMF may not be valid under low sampling rates. Consider the continuous-time signals, the definition of IMF seems intuitive and reasonable. In practical applications, signals are always sampled and presented in digital time series. According to the second condition of IMF, EMD requires the upper and lower envelopes of an IMF to be symmetric [2]. However, under low sampling rates, the upper and lower envelopes of a mono-component signal cannot be guaranteed to be symmetric. As a consequence, errors inevitably occur. For more detailed information about the sampling influence, one can refer to Rilling's work [26]. In order to deal with the low sampling problem, interpolation (or up-sampling) [27,28] is employed. If EMD is implemented without up-sampling, the sampling frequency is required to be 5 times higher than the Nyquist frequency [29].

2.3. Local narrow-band signal

Since IMF has the aforementioned shortcomings, it seems that the definition of IMF still requires further development. A substitution for the mono-component signal is the local narrow-band signal, which is able to provide a meaningful Hilbert spectrum [30]. The definition of local narrow-band is intrinsically related to the instantaneous bandwidth. If the local instantaneous bandwidth is small enough, then we can describe the signal as being local narrow-band. In Section 5.1, we will discuss in detail how to determine a local narrow-band signal. To obtain a local narrow-band signal, TVF is used in this paper. By successively applying the filter and filtering out the LLF component, the obtained LHF signal is local narrow-band.