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Stochastic stability of modified extended Kalman filter over fading channels with transmission failure and signal fluctuation

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ABSTRACT

The observations of nonlinear systems, exposed to a fading channel, greatly suffer from both transmission failure and signal fluctuation. This paper focuses on the design-oriented analysis of nonlinear estimator based on a modified extended Kalman filter (MEKF) over fading wireless networks. Bernoulli process and Rayleigh fading are taken into consideration to model transmission failure and signal fluctuation, respectively. The offline sufficient conditions are established for the boundedness of the expectations of the prediction error covariance matrices sequence (PECMS) of the MEKF, which shows the existence of a crucial arrival rate. Furthermore, based on the derived upper bound of PECMS, further sufficient conditions are provided for mean-square bounded estimate error of the MEKF using the fixed-point theorem. Numerical examples are also given to verify the analytical results and demonstrate the feasibility of the proposed methods.

1. Introduction

Over the past decade, wireless sensors and actuators have received a lot of attention due to their advantages of low-costs and easy of expansion [1,2]. Modern industrial systems are widely equipped with wireless sensors, bringing high requirements for the monitoring systems [3–6]. The most effective way to realize the monitoring is by means of state estimation using linear Kalman filter (KF) and nonlinear filter, i.e., Kalman variants, which catalyzes the development of nonlinear filter because the nonlinear system is the overwhelming majority in practice [7–12]. The stochastic stability of the filter is a necessary condition to guarantee the effectiveness of monitoring systems.

However, the communication channel between wireless sets are susceptible to environmental influence. Thus, the drawback due to the wireless communication channels must be taken into consideration when conducting the analysis of the stochastic stability and performance of the filter. The major constraints of wireless channel, reducing the estimation performance, are the transmission bandwidth and fading channel [13]. On one hand, the filter under the band-limited channel is confronted with the challenges of the time-delay and quantization effect, which has been profoundly studied by Shi et al. [14], Wu and Wang [15], Su et al. [16,17] and Caballero-Águila et al. [18]. On the other hand, the fading channel also causes unstable issues to the filter so that the stochastic stability and performance analysis of filter under fading channel becomes indispensable for the estimation systems design [19,20].

The crucial factor for an effective analysis of practical estimator is the modeling precision of the fading channel. Some research utilizes a binary treatment for the receiving information through the fading channel, which either trusts the information and utilizes it as an observation or drops it as a transmission failure. Such filtering under that channel structure was named as the filter with intermittent observations, and the stochastic stability of KF with intermittent observations for linear time invariant (LTI) was firstly studied by Sinopoli et al. [21]. That work pointed out that the prediction error covariance matrices sequence (PECMS) of the filter was random rather than deterministic, and the expectation of PECMS was exponentially bounded if the arrival rate exceeded a critical probability when the arrival of the observations conformed to a Bernoulli process. By utilizing more complicated channel model to describe the transmission failure, i.e., the Gilbert-Elliott channel model and finite state Markov process, a variety of research extended the analysis of filter with intermittent observations to more general application scenario [22–25]. Moreover, some research extended the work from LTI system to nonlinear system [26–29].

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The other modeling method of fading channel is to describe the effectiveness of the observation information by the signal to noise ratio (SNR), which is also named as signal fluctuation [19,30,31]. It was pointed out that PECMS of the filter was a random variable because the signal fluctuation introduced randomness into channels, and upper bounds of means of PECMS was deduced if the channel's SNR followed the specified distribution [30]. Besides, Quevedo further proposed that SNR was highly related to the channel gain, which was determined by the engineering parameters, i.e., the bit-rate and power level [19,31]. Furthermore, the performance analysis was extended from the filter level to the whole wireless estimation system level in [19,31,32]. Among all these works, the filter performance was analyzed in [33] and [34] under Rayleigh fading channel by verifying the upper error outage probability, which was of practical importance.

Because both the modeling methods for fading channel are reasonable, an unified consideration about both the transmission failure and signal fluctuation is able to handle more comprehensive problems introduced by the practical problem from channels [20,35]. KF for LTI system with both transmission failure and signal fluctuation was taken into consideration in [35], where the sufficient and necessary conditions for the stochastic boundedness of PECMS were put forward by the modified Lyapunov and Riccati iteration methods, respectively. The work was further extended to the time-varying KF with more generated fading model, where the transmission failure of channel was described as a Markov chain [20]. In the case of the nonlinear system, an UKF based filter with both disturbances was studied in [36].

Similar to [35], the mean convergence of the PECMS was studied and an upper bound sequence for the PECMS of UKF was given. However, PECMS is a significant criterion of KF for linear systems because the estimation error is a zero mean Gaussian vector with the covariance matrix equal to the PECMS. On the contrary, it becomes unsuitable for nonlinear system only in terms of the stability and performance of PECMS so that the mean-square estimation error is the proper indicator for nonlinear system. Moreover, the unknown diagonal matrix similar to [27], which was a part of the parameters to calculate the upper bound, made the theorem in [36] difficult in application as an off-line analysis method for general nonlinear system.

Motivated by these concerns for nonlinear systems, it is of significant necessity to study off-line sufficient conditions for the stochastic boundedness of PECMS and estimate error with both transmission failure and signal fluctuation. This effort can be utilized to design and analyze the fusion estimator over fading wireless networks. This paper focuses on the MEKF over fading channel with both disturbances and off-line sufficient conditions are established for the boundedness of both the mean of PECMS and the mean-square of the estimate error. Because the sufficient conditions for the boundedness of estimate error contain the relationship between the upper mean bound of PECMS and the system Jacobi matrix, an upper bound sequence for the mean of PECMS is also proposed in this paper.

The rest of this paper is organized as follows. Section 2 introduces the nonlinear system and the fading channel which the observations are transmitted through. Moreover, the MEKF is established based on EKF and a proposed drop strategy. In Section 3, it is proved that there exists a critical value λ_c. If the arrival rate $\tilde{\lambda} > \lambda_c$ is guaranteed, the mean of the PECMS (i.e., $\mathbb{E}[\hat{P}_{t+1|t}]$) will be bounded for all initial conditions. In Section 4, an explicit expression sequence is proposed as the upper bound of the PECMS. Section 5 further derives the sufficient off-line conditions for the boundedness of $\|e_{t|t-1}\|$ based on the upper bound of the PECMS. Section 6 conducted various numerical simulations to verify the theorems in previous sections.

The following standard notations are adopted throughout this paper. The norm of vector $\|x\|$ stands for the Euclidian norm, and the norm of matrix $\|A\|$ stands for the spectral norm. $\mathbb{E}(x)$ denotes the expectation value of x, and the $\mathbb{E}(x|y)$ denotes the expectation value of x conditional on y. I_n stands for the identity matrix with dimension n, and the I stands for the identity matrix with the suitable dimension. The matrix $\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$ is shortened as $A_1 \oplus A_2$. Finally, $x \sim \mathcal{N}(\bar{x}, P)$ express that x follow the Gaussian distribution with $\bar{x}$ mean and P covariance.

2. Problem statement

Consider the discrete time nonlinear dynamical system:

$$\begin{aligned} x_{t+1} &= f(x_t) + \omega_t, \\ \bar{z}_t &= h(x_t) + \nu_t, \end{aligned} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state and $\bar{z}_t \in \mathbb{R}^p$ is the measured output. The system function $f(x)$ and estimate function $h(x)$ are continuously differentiable at every x. The process noise $\omega_t \in \mathbb{R}^n$ and measurement noise $\nu_t \in \mathbb{R}^p$ are both white Gaussian noise with the covariance matrices $Q > 0$ and $R > 0$, respectively. It is assumed that the initial state x_0 is also Gaussian random vector with the covariance matrix R_0. Moreover, ω_t,ν_t and x_0 are independent with each other. The measurement $\bar{z}_t$ is transmitted over a wireless fading channel with both fluctuant and transmission failure.

2.1. Effects of channel fading with transmission failure and signal fluctuation

In this part, the impact of a time-varying fading communication channel will be modeled on the observation. Let $\bar{z}_t$ and z_t represent the measurement in system (1) and the received observation of filter, respectively. The model of fading channel with both fluctuation and transmission failure is thus given by Xiao et al. [35]:

$$z_t = \xi_t \bar{z}_t + \eta_t, \tag{2}$$

where $\eta_t \in \mathbb{R}^p$ is the channel additive noise, which is white Gaussian noise with covariance matrices $\Xi > 0$. $\xi_t \in \mathbb{R}$ represents the fading channel, which consists of transmission failure and gain fluctuation, i.e.,

$$\xi_t = \gamma_t \vartheta_t. \tag{3}$$

The change gain ϑ_t is caused by the fluctuation, whose most common statistical model is Rayleigh fading. If $\iota_t = \vartheta_t^2$, by the property of Rayleigh fading with the parameter ϵ, ι_t is white and its distribution is that, $\iota_t \sim \epsilon \exp(-\epsilon \iota_t)$. The arrival of the observation at time t is defined as a binary random γ_t:

$$\gamma_t = \begin{cases} 1 & \text{the filter successfully get the observation} \\ 0 & \text{the observation suffers from the transmission failure.} \end{cases} \tag{4}$$

γ_t is a Bernoulli process with the parameter λ, which means that γ_t is a sequence of independent identically distributed with the arrival rate $\mathbb{P}\{\gamma_t = 1\} = \lambda$ [21].

The observation function of discrete-time nonlinear dynamical system together with the time-varying fading communication channel can be written as:

$$z_t = \gamma_t \vartheta_t h(x_t) + \gamma_t \vartheta_t \nu_t + \eta_t, \tag{5}$$

where x_0, ω_t, ν_t, η_t, γ_t and ϑ_t are uncorrelated with each other.

Remark 1. By the help of time-stamped technology, the information of γ_t together with the observation z_t are available for the filter at time t. It is assumed that the channel gain ϑ_t is valid for the filter at time t by the wireless communication technology in [37]. Also, it could be supposed that the channel gain remains constant during the transfer of the tth data, which is suitable when